Fundamentals of GNSS Processing

GNSS4SWEC Summer School, Bulgaria, 2014

Hans van der Marel, TU Delft

8 September 2014

Outline

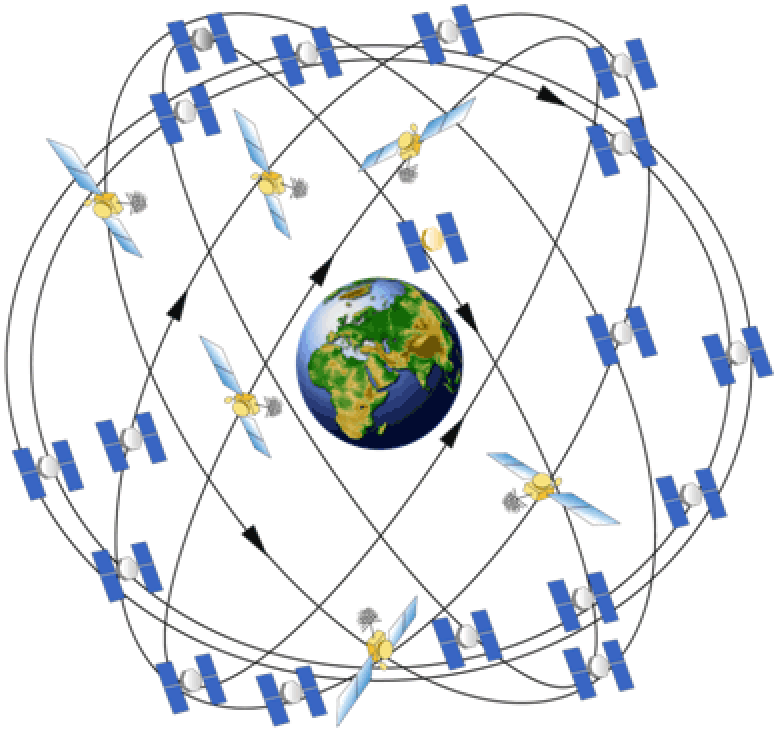
- GNSS Systems
- GPS – How it works
- Pseudo-range measurement
- Carrier phase measurement
- Standard GNSS navigation solution
  - →Found in every receiver
  - →We can (must) do better than this for Earth Sciences
- GNSS error budget
  - What to do about it…?
- GNSS Carrier Phase Solution
- Integrated Water Vapour (IWV) Processing
- Differential (Relative) GNSS Baseline and Network Solutions
- International GNSS Service
- …

Fundamentals of GNSS Processing

GNSS SYSTEMS

GNSS Overview

- Global Navigation Satellite System (GNSS) *)
  - → GPS (U.S.A.)
  - → Galileo (Europe)
  - → GLONASS (Russia)
  - → BEIDOU (China)
- Generic term for any system that provides autonomous geo-spatial positioning with global coverage
- Each system has 3 basic components
  - → Space segment (satellites)
  - → Control segment (maintenance and service)
  - → User segment (receivers, etc.)

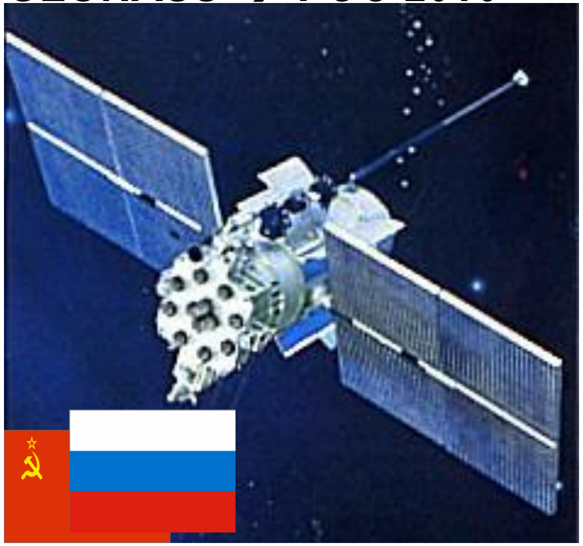


*) regional systems IRNSS,QZSS not included in overview.

GNSS =
Global
Navigation
Satellite
System

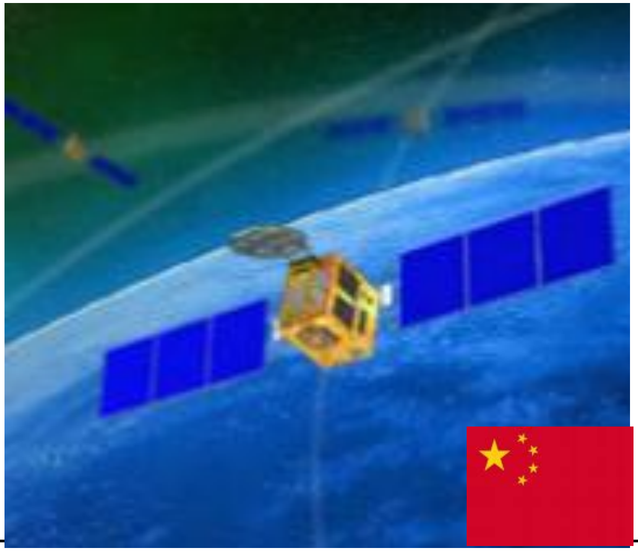
GPS → *FOC 1995*

GLONASS → *FOC 2010*



GALILEO → *FOC 2019?*

BEIDOU → *FOC 2020*



FOC=Full Operational Constellation

GNSS Satellite Orbits

In 2020 we could have 100 satellites

Number of satellites: #nominal (#operational)

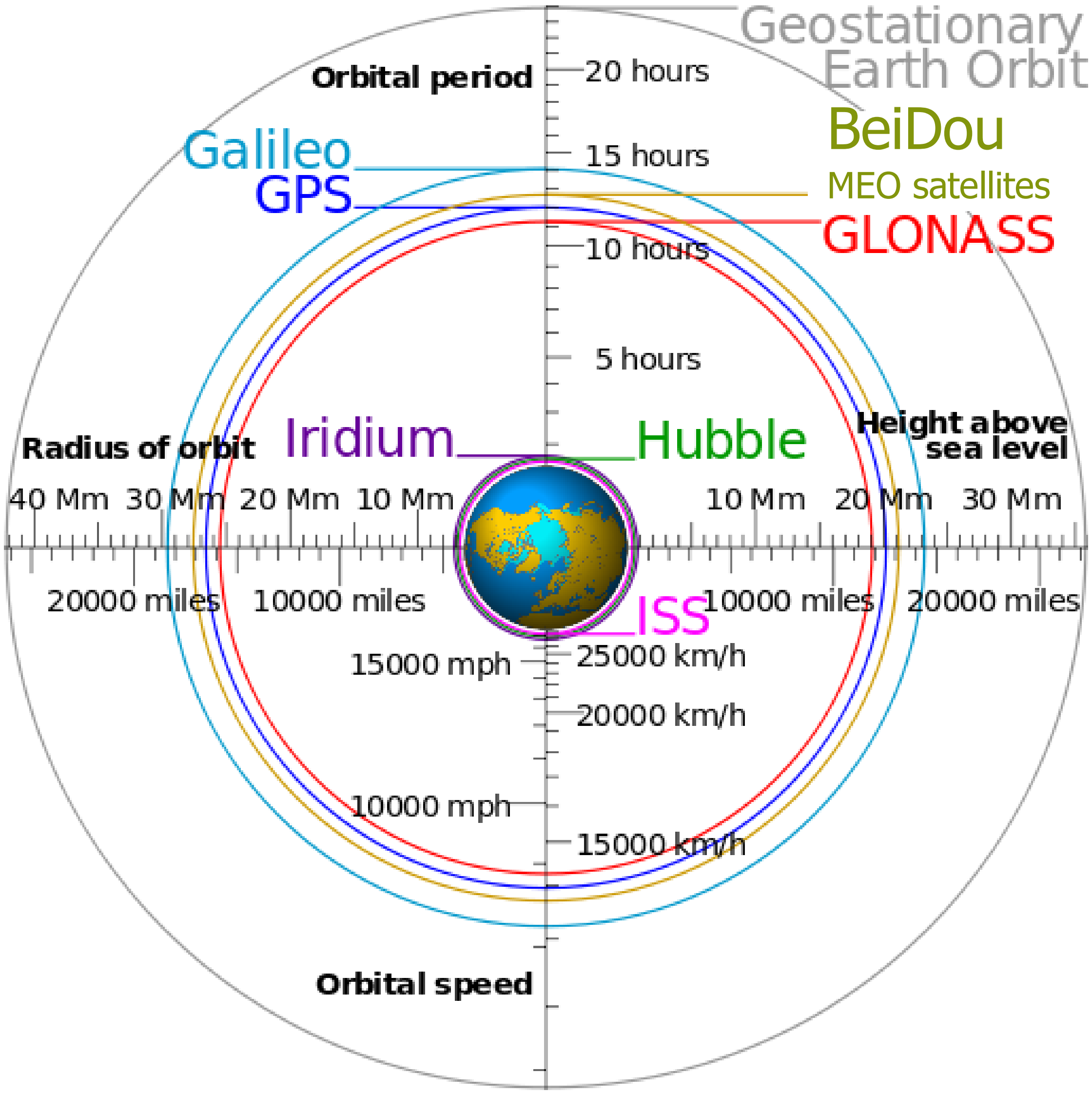
	GPS	GLONASS	BeiDou	Galileo
# MEO	24 (30)	24 (24)	27 (4)	30 (6)
# IGSO			3 (5)	
# GEO	WAAS/EGNOS		5 (5)	EGNOS-2

MEO orbit parameters:

	GPS	GLONASS	BeiDou	Galileo
Orbital height	20,180 km	19,130 km	21,150 km	23,220 km
Orbital period	11 h 58 m	11 h 16 m	12 h 38 m	14 h 5 m
Revs/siderial_day	2	17/8	17/9	17/10
Inclination	55^o	64.8^o	55^o	56^o

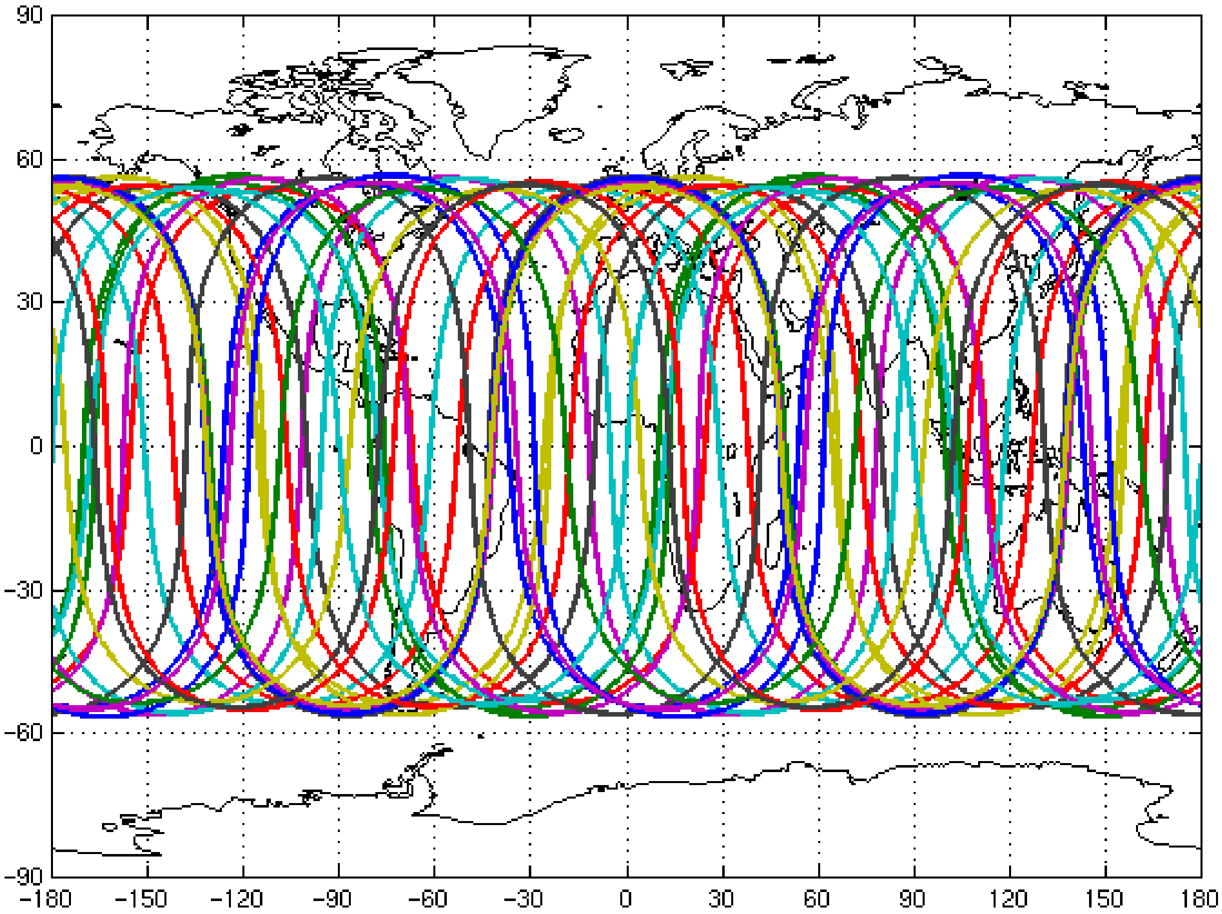
MEO = Medium Earth Orbit
IGSO = Inclined Geo Synchronous Orbit
GEO = Geostationary Earth Orbit

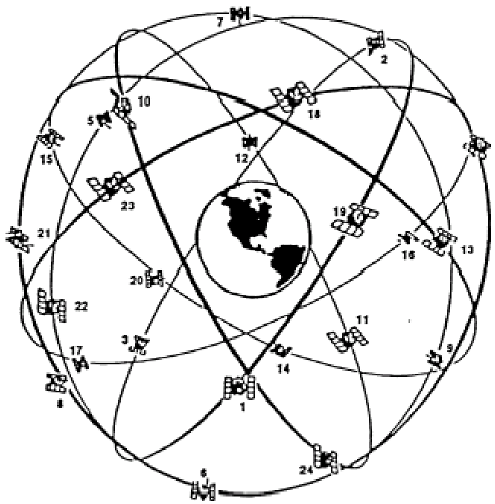
WAAS, EGNOS, … are regional augmentation systems.
IRNSS,QZSS not included in overview.



© Adapted from Wikipedia

GPS Satellite Ground Tracks





nearly circular orbits

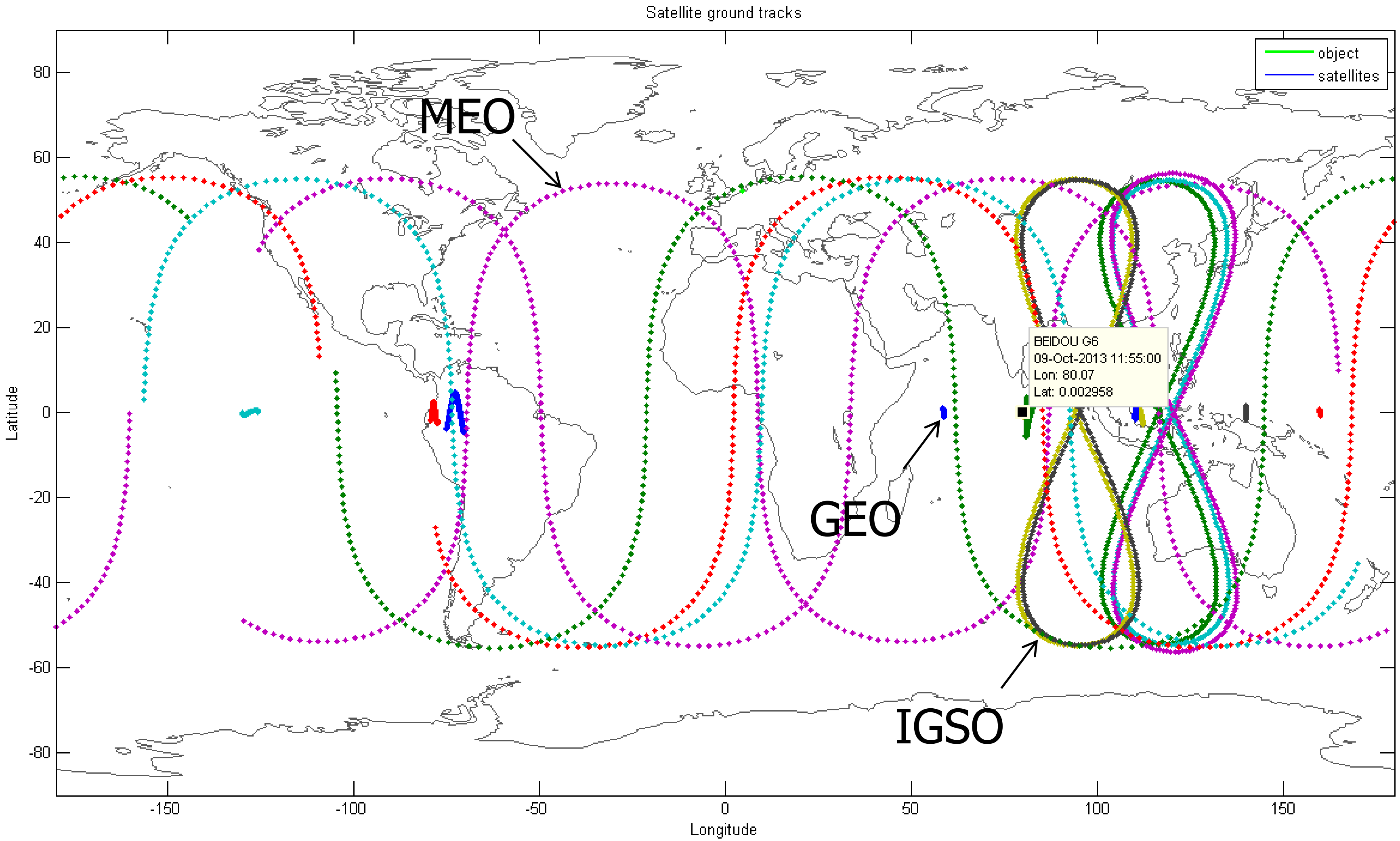
orbital period 11^h58^m

20,200 km altitude

inclination 55^0

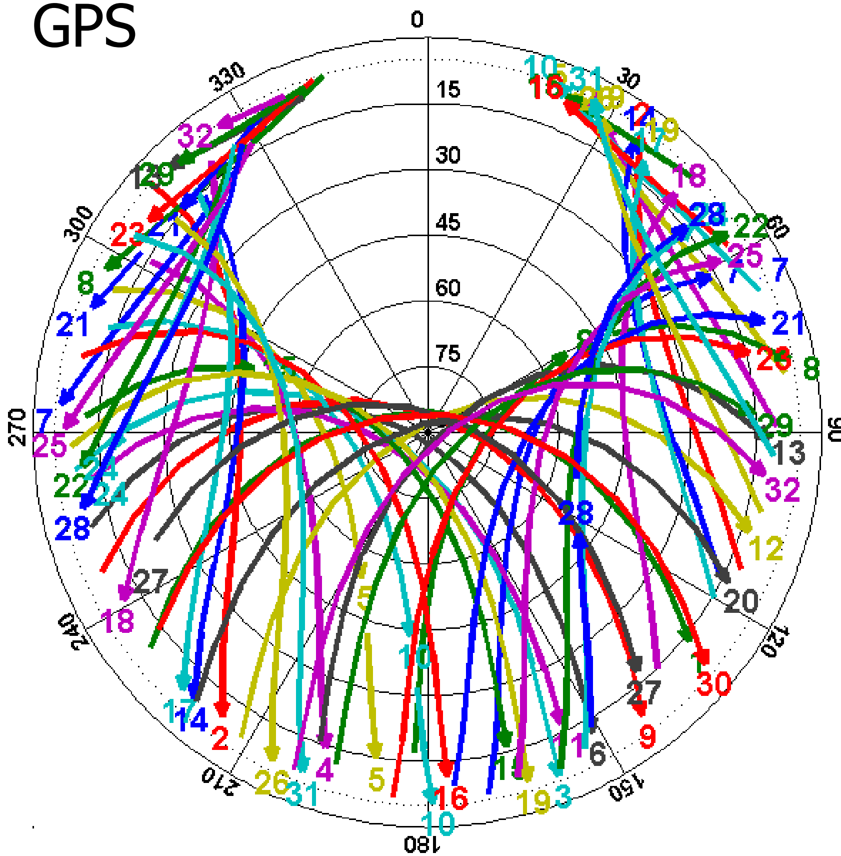
6 orbital planes

BeiDou Ground Tracks (9 Oct 2013)



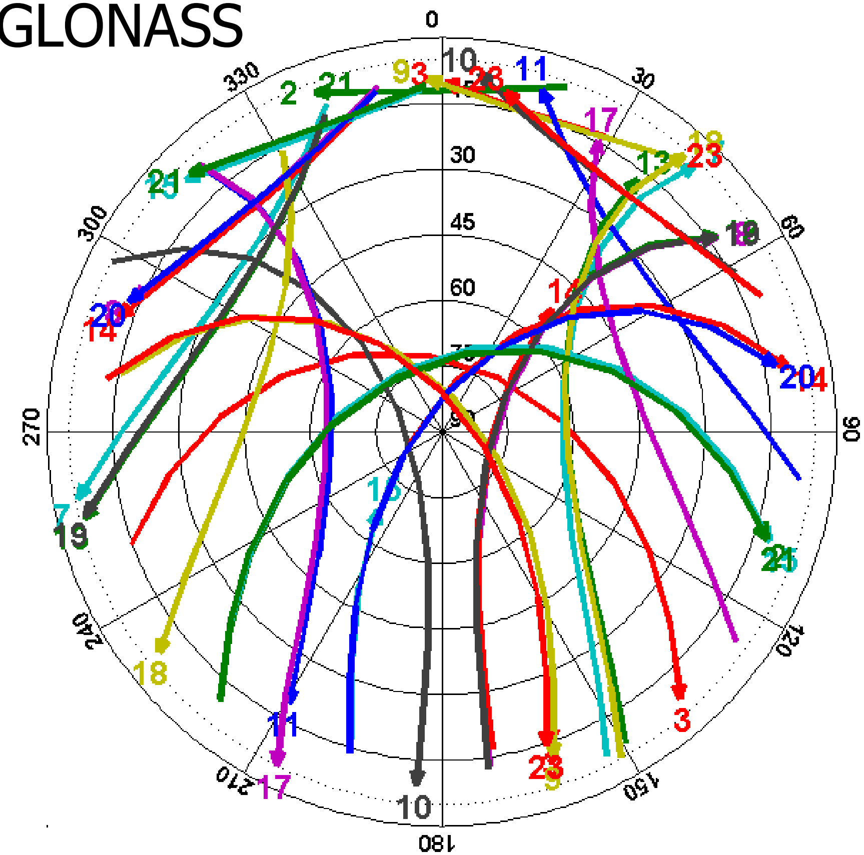
GPS and GLONASS Skyplots (Delft)

GPS



Repeats daily (2 revs/siderial day)

GLONASS

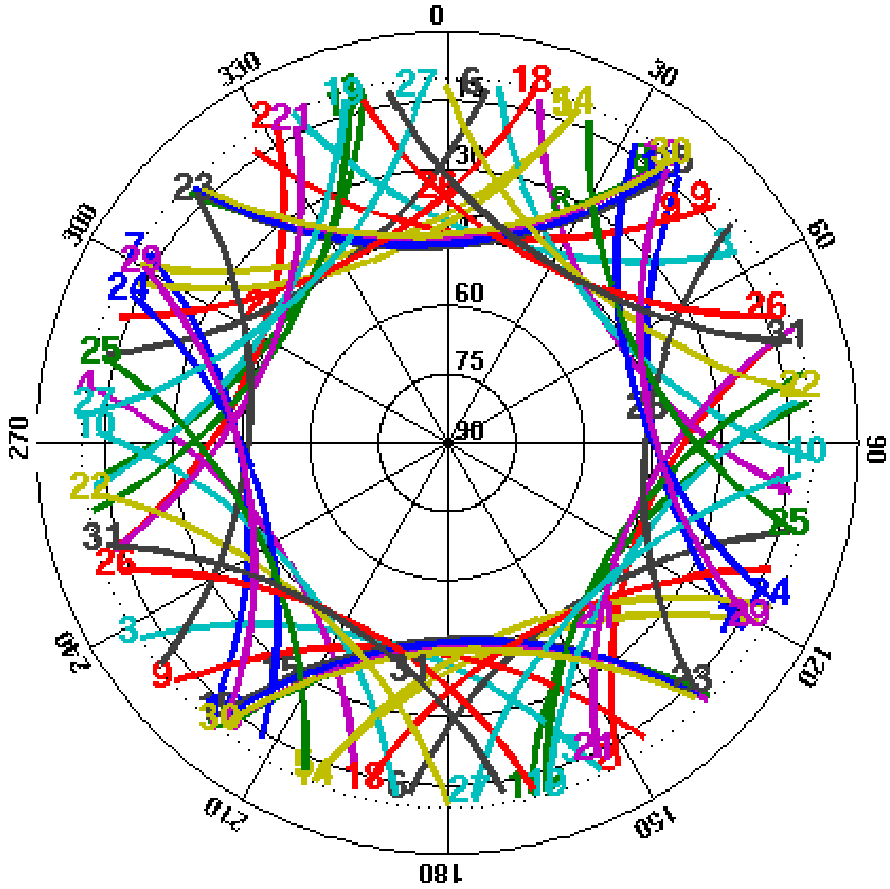


Repeats every 8 days
(17/8 rev/siderial day)

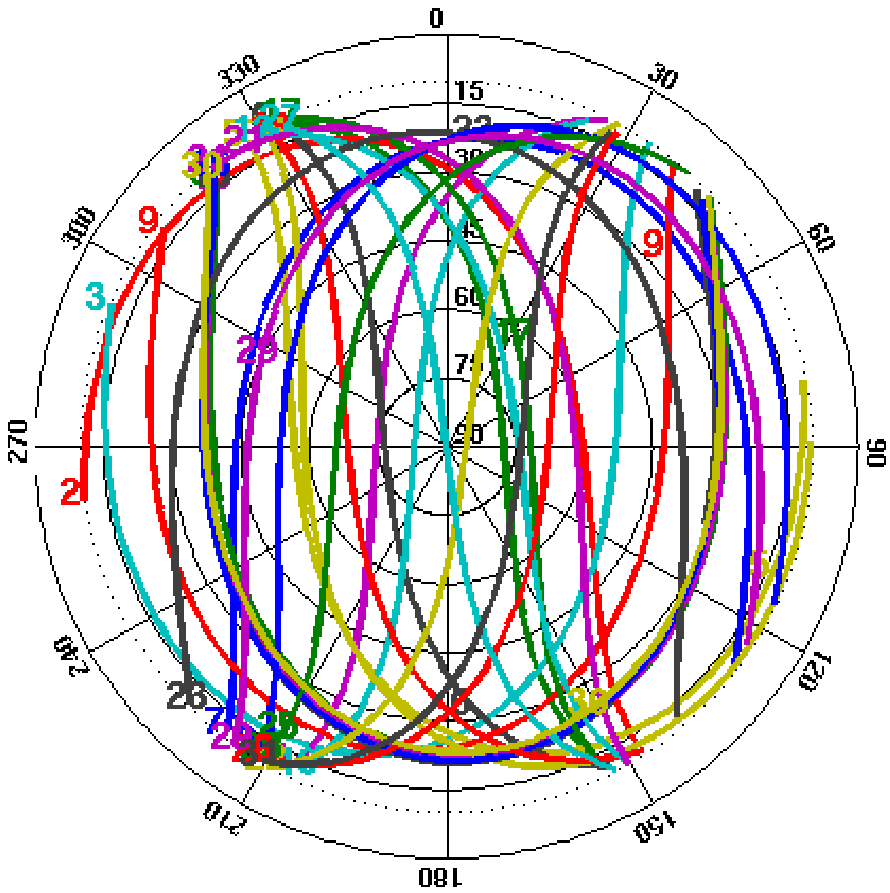
More GPS Skyplots...

1-Dec-1999, 24h period

North Pole



Equator

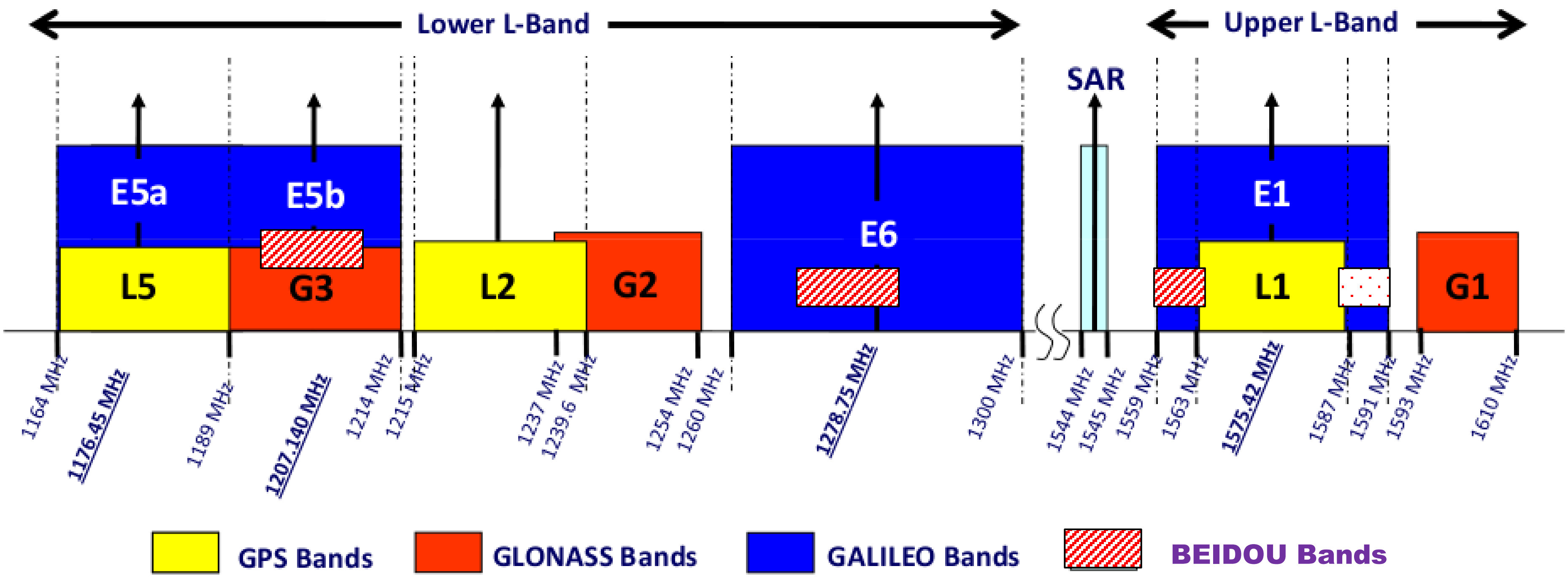


GNSS Frequencies

	GPS	GLONASS	BeiDou	Galileo
coding	CDMA	FDMA/CDMA	CDMA	CDMA
Upper L band (MHz)	1575.42 (L1)	1593-1610 (G1)	1561.098 (B1) 1589.742 (-)	1575.42 (E1)
Lower L band (MHz)	1227.06 (L2) 1176.45 (L5)	1237-1254 (G2) 1207.14 (G3)	1268.52 (B3) 1207.14 (B2)	1278.75 (E6) 1207.14 (E5b) 1176.45 (E5a)
status	is modernized	CDMA planned		

1575.42 (L1) - Fully operational signals
1207.14 (B2) - Signals in build up phase
1589.742 (-) - Not yet detected

GNSS Frequencies



but also how other GNSS systems work …

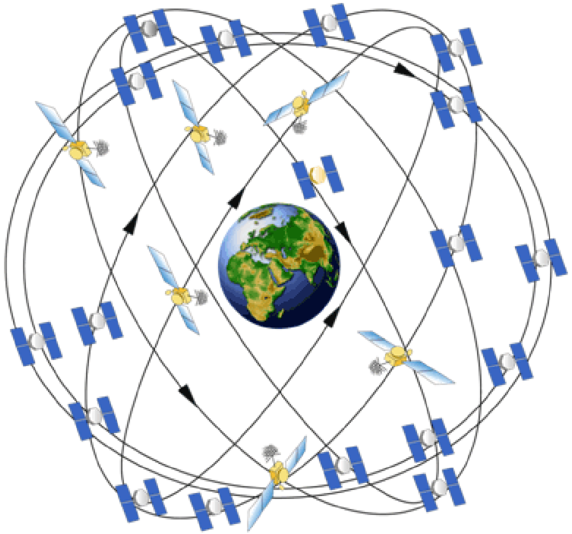
Fundamentals of GNSS Processing

GPS – HOW IT WORKS

GPS System – how it works

Space segment

- 30 spacecraft (24 nominal)
- 20200 km altitude (11:58 orbit)
- Spread across 6 orbit planes
- Inclination 55^{o}

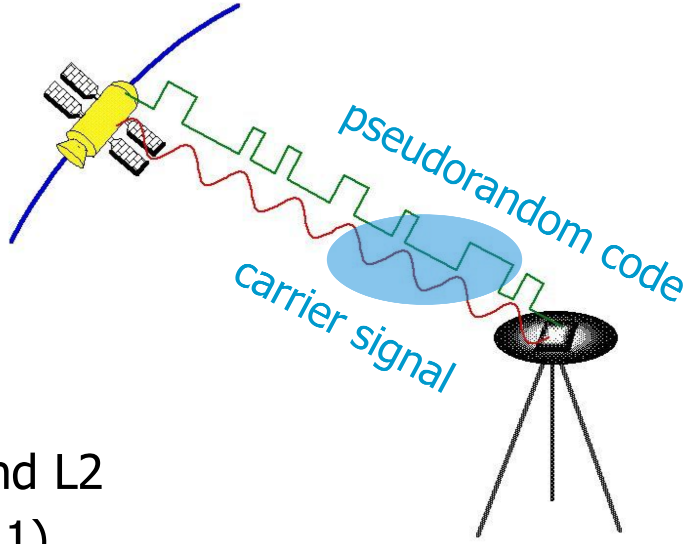


Control segment

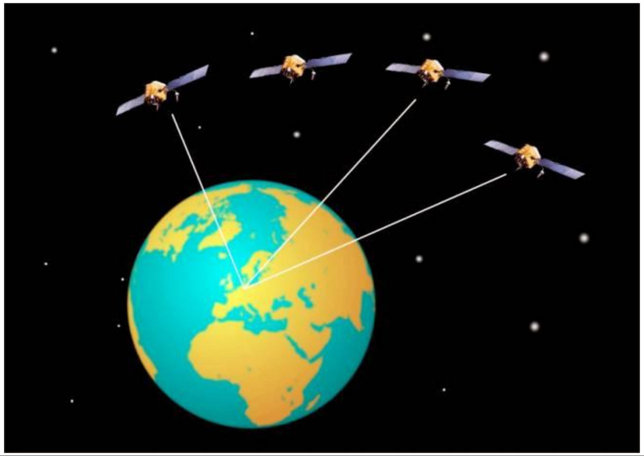
- Tracking stations
- Satellite ephemerides computation
- Upload of ephemerides to satellites

GPS System – how it works

- Signal transmitted on two frequencies
  - ➔ f_0 = 10.23 MHz (fundamental freq)
  - ➔ L1 = 1575.42 MHz = $154*f_0$
  - ➔ L2 = 1227.60 MHz = $120*f_0$



- Pseudo Random Noise (PRN) code
  - ➔ Additional signals super-imposed on L1 and L2
  - ➔ C/A = Coarse/Acquisition code (only on L1)
    - ❑ Repeats every millisecond
    - ❑ Unique code for each GPS satellite
    - ❑ Can get position to within ~5m
  - ➔ P-code, A/S

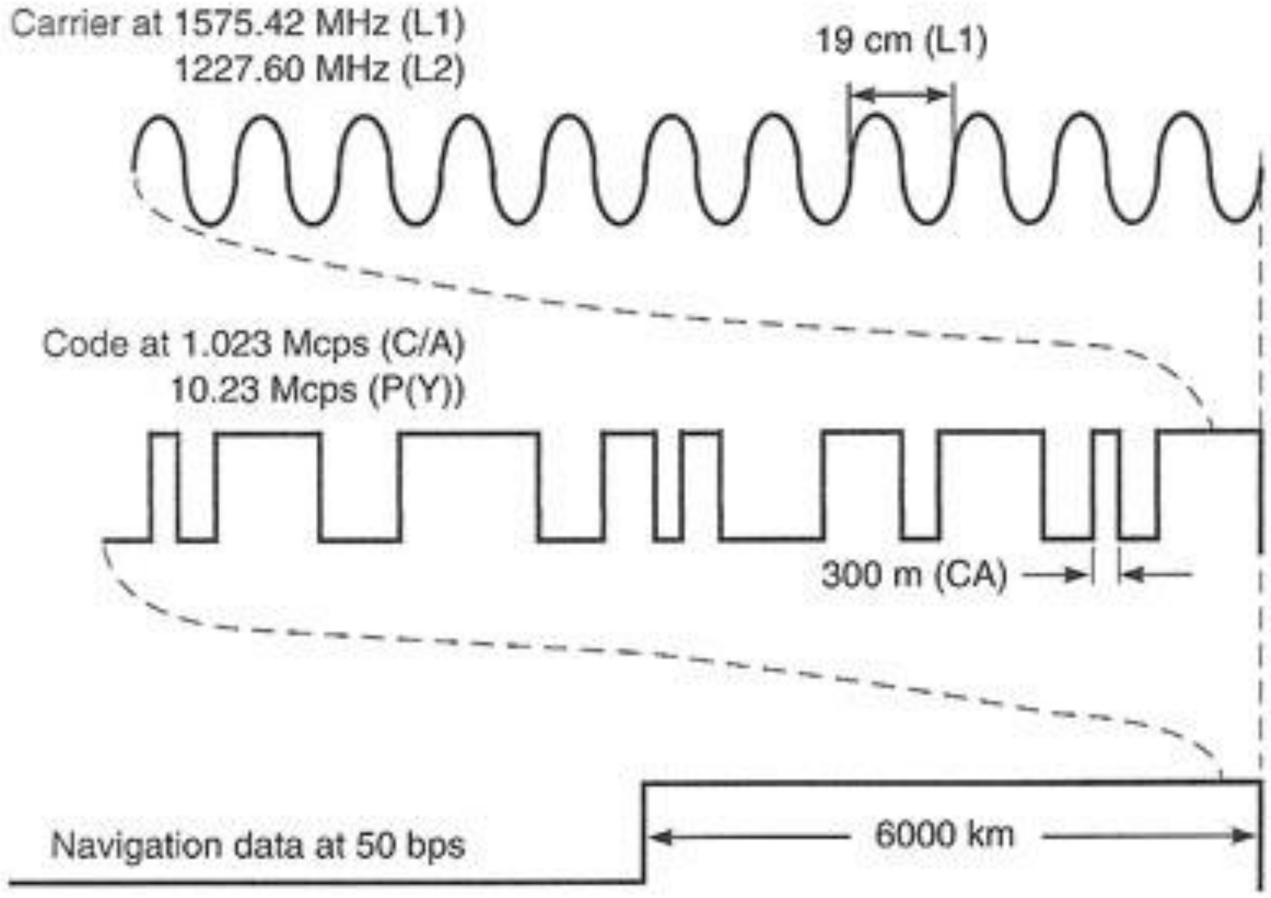


- Satellite ephemeris in data message

GPS System – how it works

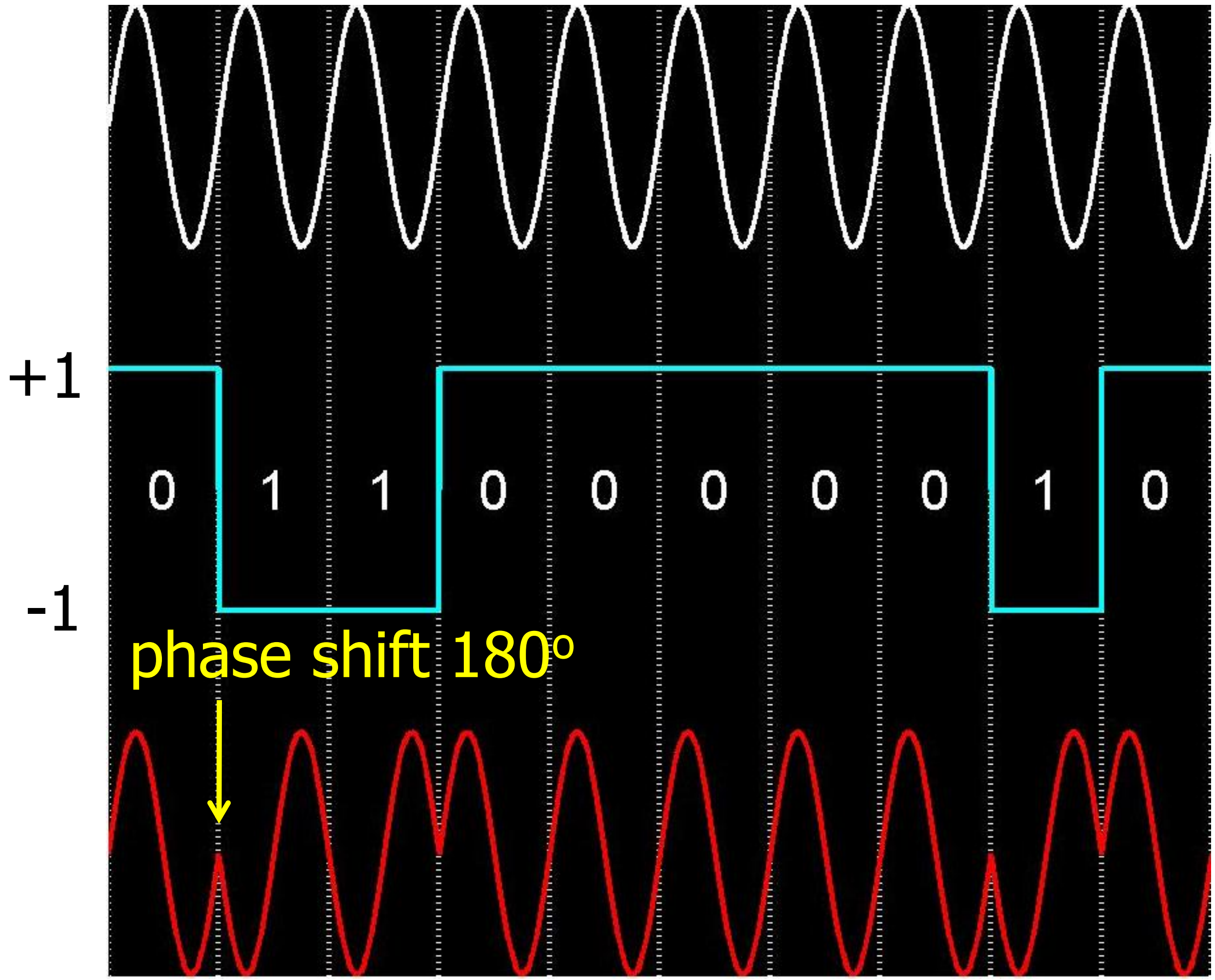
- Receiver identifies each GPS sat signal through PRN (code division multiple access)
- Position of GPS satellites known through broadcast ephemeris (updated every 4 hours)
- Travel time of signal determined by comparing received and internally generated PRN codes
  - →More accurate methods measure phase of signals
- Receiver clock often not that accurate, so need to track an additional satellite to estimate clock error
  - →Need at least 4 GPS satellites in total to determine position

GPS signal components



From: Misra and Enge

Binary Phase Shift Keying (BPSK)

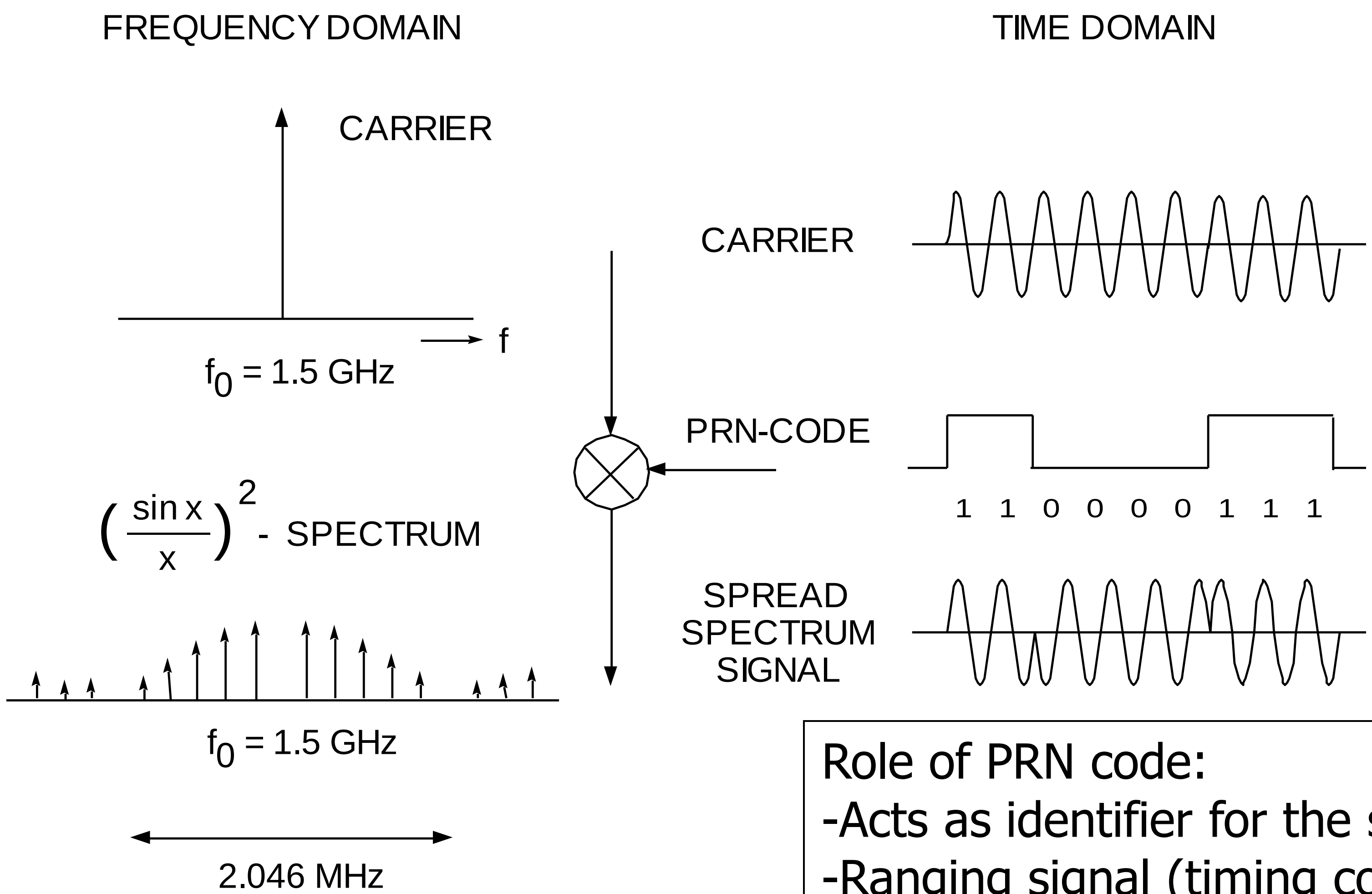


carrier
$f(t)$

code and data
$C(t) \oplus D(t)$

signal
$S(t)$

Binary Phase Shift Keying (BPSK)



Role of PRN code:
- Acts as identifier for the satellite
- Ranging signal (timing code)

PRN codes

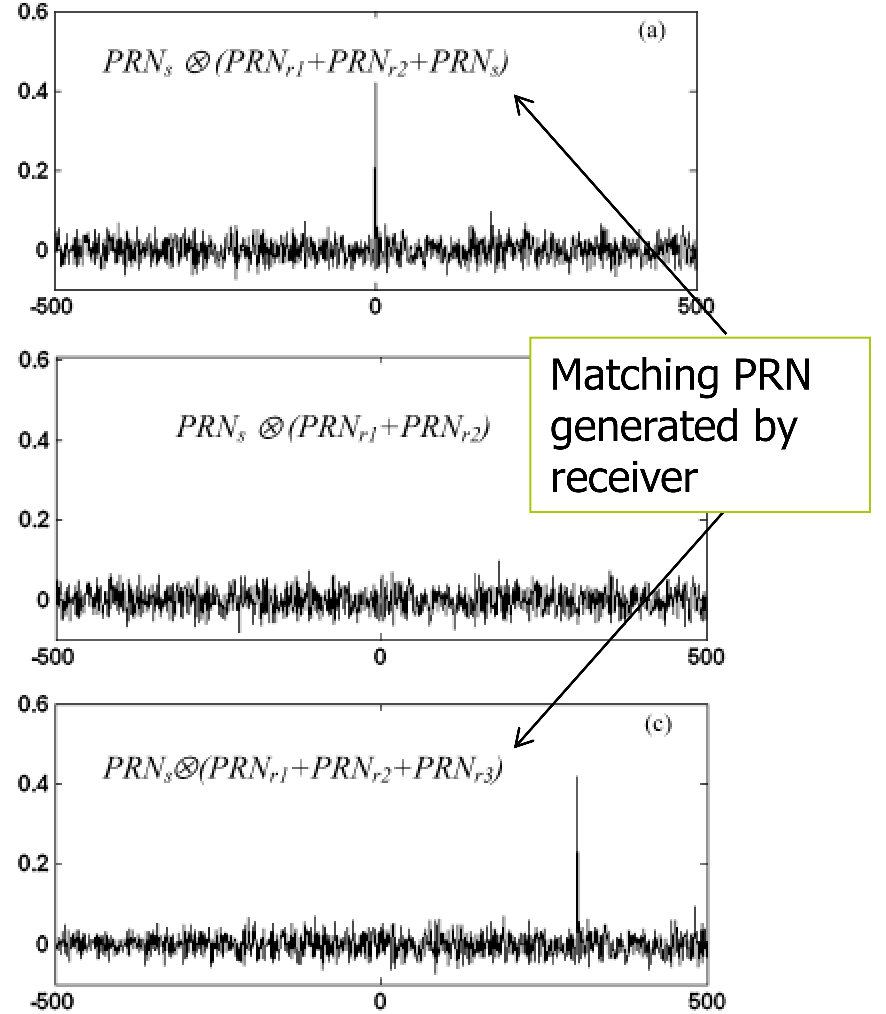
Two functions:
- Time signal => pseudo-range
- Identification of satellite

Both functions achieved through auto correlation

$$\Psi(\tau) = \frac{1}{T}\int_0^T f_1(t) f_1(t-\tau)\,dt$$

$$\Psi_c(\tau) = \frac{1}{T}\int_0^T f_1(t) f_2(t-\tau)\,dt$$

PRN codes have high auto-correlation and low cross-correlation



[Abid, 2005]

GPS signal components

- All signals and time information are coherently derived from the same clock with a frequency of **f_0=10.23 MHz**

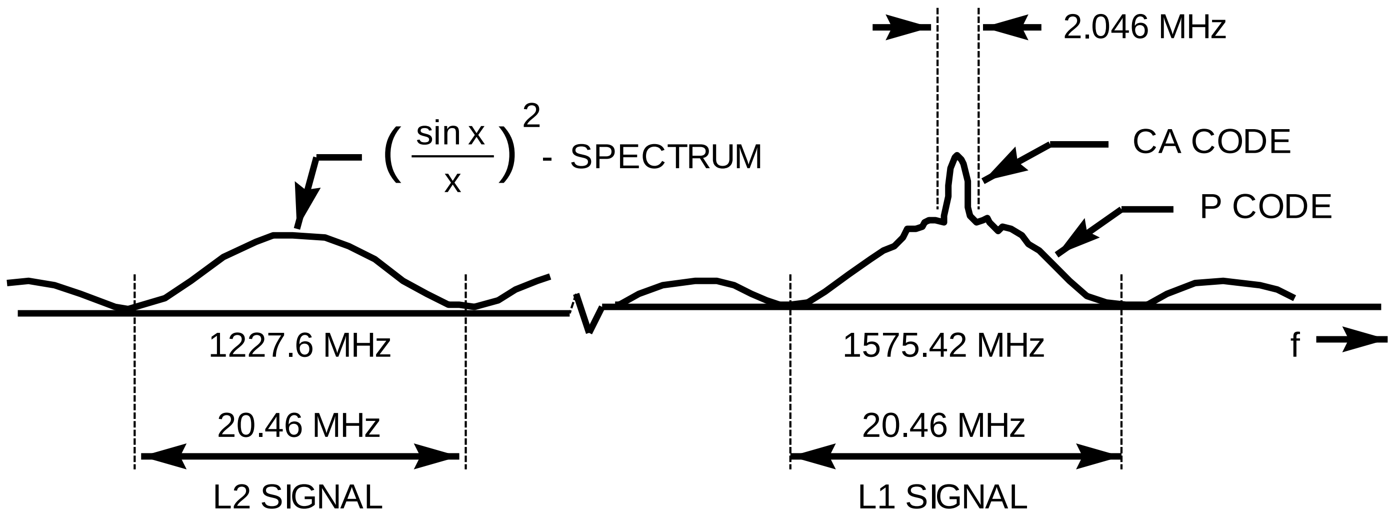
Signal components	Frequency	Wavelength / chiplength
L1 carrier	1575.42 MHz (154*f_0)	19.05 cm
L2 carrier	1227.60 MHz (120*f_0)	24.45 cm
C/A code on L1	1.023 Mbits/sec (0.1*f_0)	293 m
~~**P(Y)** code on L1 & L2~~	~~10.23 Mbits/sec (f_0)~~	~~29.3 m~~
Broadcast message	50 bits/sec	

In future: additional civil codes on L1, L2 and L5

Y-code is encrypted version of P-code (using encryption code W)

GPS Spread-Spectrum signal

- BPSK modulation with a PRN code sequence produces a broad bandwidth
  This is why it is also called spread spectrum modulation
  - Limits the interference from other signals
  - Chosen by the U.S. military to prevent jamming and spoofing
- Spectrum is a sinc function



- GPS signals are very weak when received (below the noise) how to measure GNSS satellite ranges

a.k.a. code (phase) measurement

Fundamentals of GNSS Processing

PSEUDO-RANGE MEASUREMENT

Pseudo-range measurement

τ	transit time	→ 70 to 90 ms
t	**true** GPS time at which code is received	
$t^s(t-\tau)$	emission time (imprinted on signal)	
$t_r(t)$	arrival time (Rx clock reading)	

$$\rho(t) = c\left[t_r(t) - t^s(t-\tau)\right]$$

Pseudo-range

Receiver clock bias

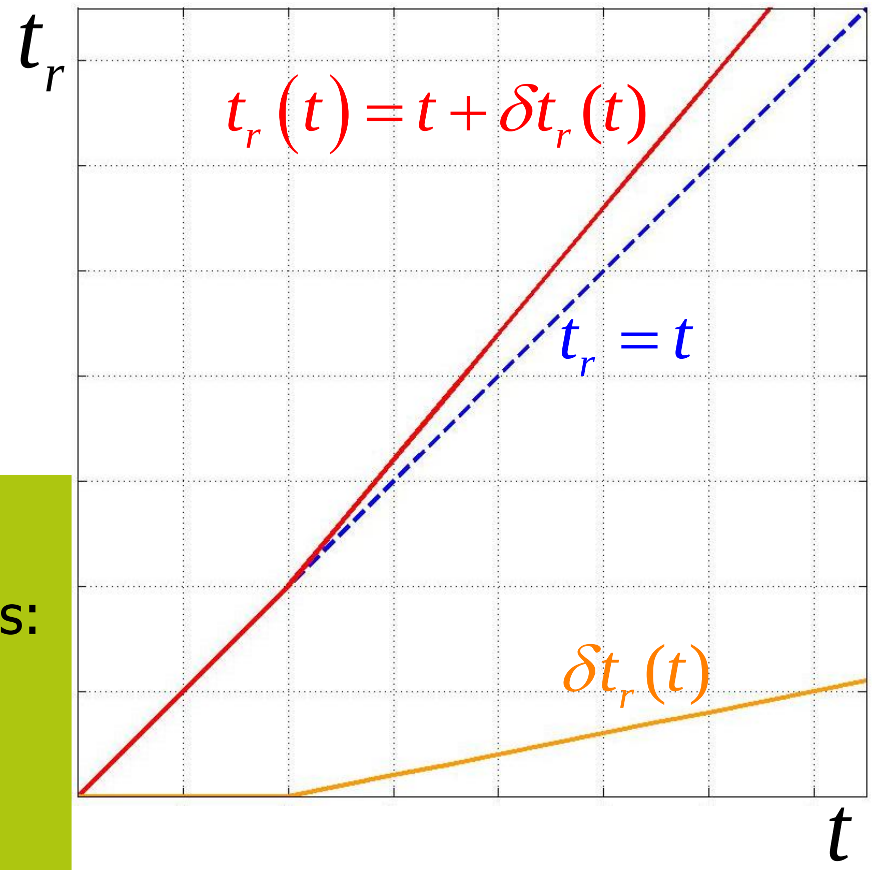
$$t_r(t) = t + \delta t_r(t)$$

receiver clock bias

Receiver clocks: drift!

Deviation from GPS time limited to ±1 ms:

- continuous clock steering, or,
- reset (clock jump!) when certain threshold is reached



Satellite clock bias

$$t_r\left(t\right)=t+\delta t_r(t)$$

receiver clock bias

$$t^s\left(t-\tau\right)=(t-\tau)+\delta t^s(t-\tau)$$

satellite clock bias

estimated by control segment

Pseudo-range observation equation

$$t_r(t) = t + \delta t_r(t)$$

$$t^s(t-\tau) = (t-\tau) + \delta t^s(t-\tau)$$

Unmodeled effects and errors

$$\rho(t) = c\left[t_r(t) - t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

$$= c\left[\cancel{t} + \delta t_r(t) - (\cancel{t} - \tau) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

$$= c\tau + c\left[\delta t_r(t) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

Pseudo-range observation equation

$$\rho(t) = c\tau + c\left[\delta t_r(t) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

distance traveled by signal

clock biases

noise + errors

Pseudo-range observation equation

$$\rho(t) = c\tau + c\left[\delta t_r(t) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

clock biases

noise + errors

distance traveled by signal

$$c\tau = r(t, t-\tau) + I_\rho(t) + T_\rho(t)$$

geometric range

ionosphere
and troposphere delays

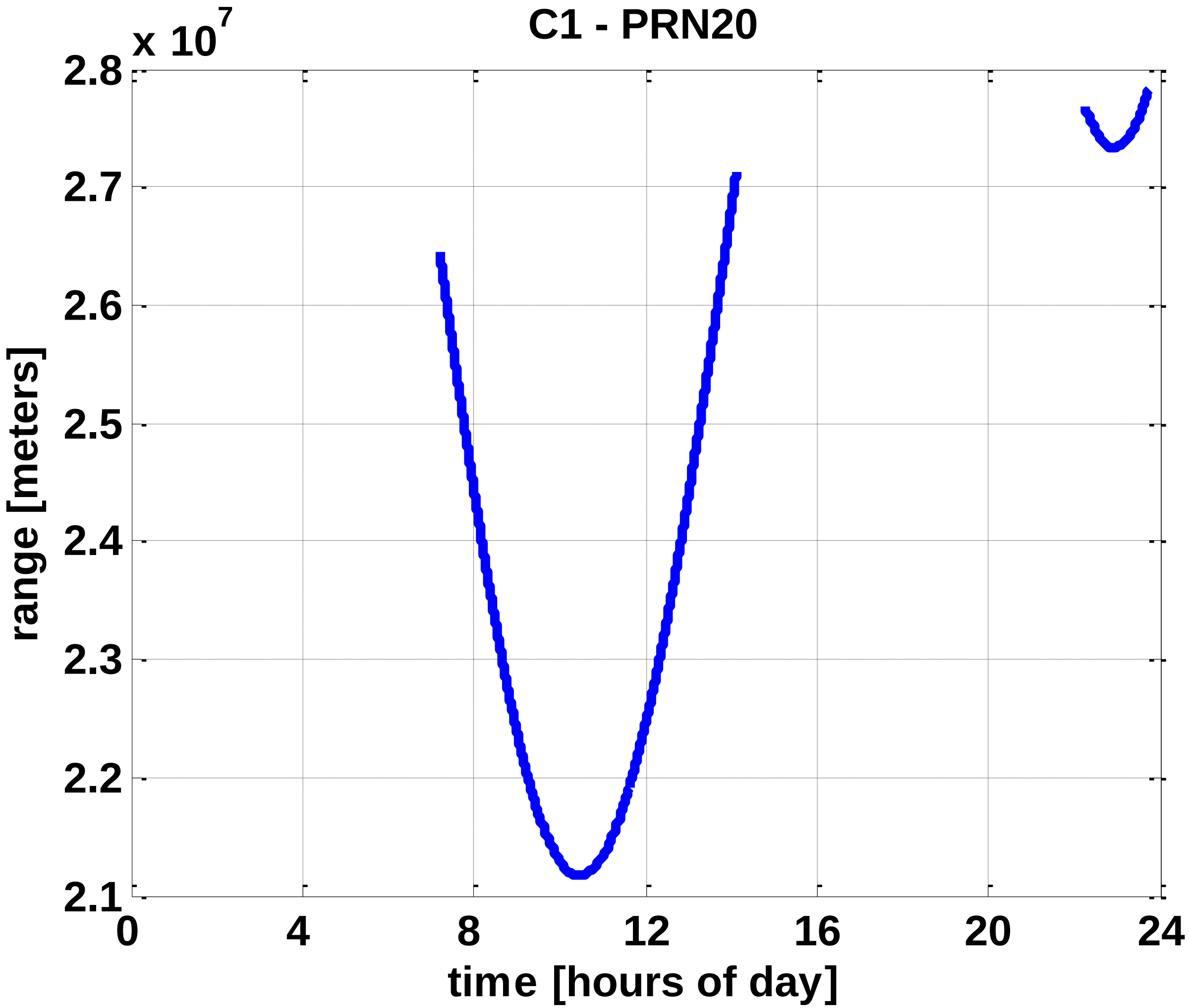
Pseudo-range observation equation

$$\rho = r + I_\rho + T_\rho + c\left[\delta t_u - \delta t^s\right] + \varepsilon_\rho$$

pseudorange measurement = biased and noisy measurement of the **geometric range *r***

This is the "geometry free" pseudo-range observation equation (for all GNSS systems)

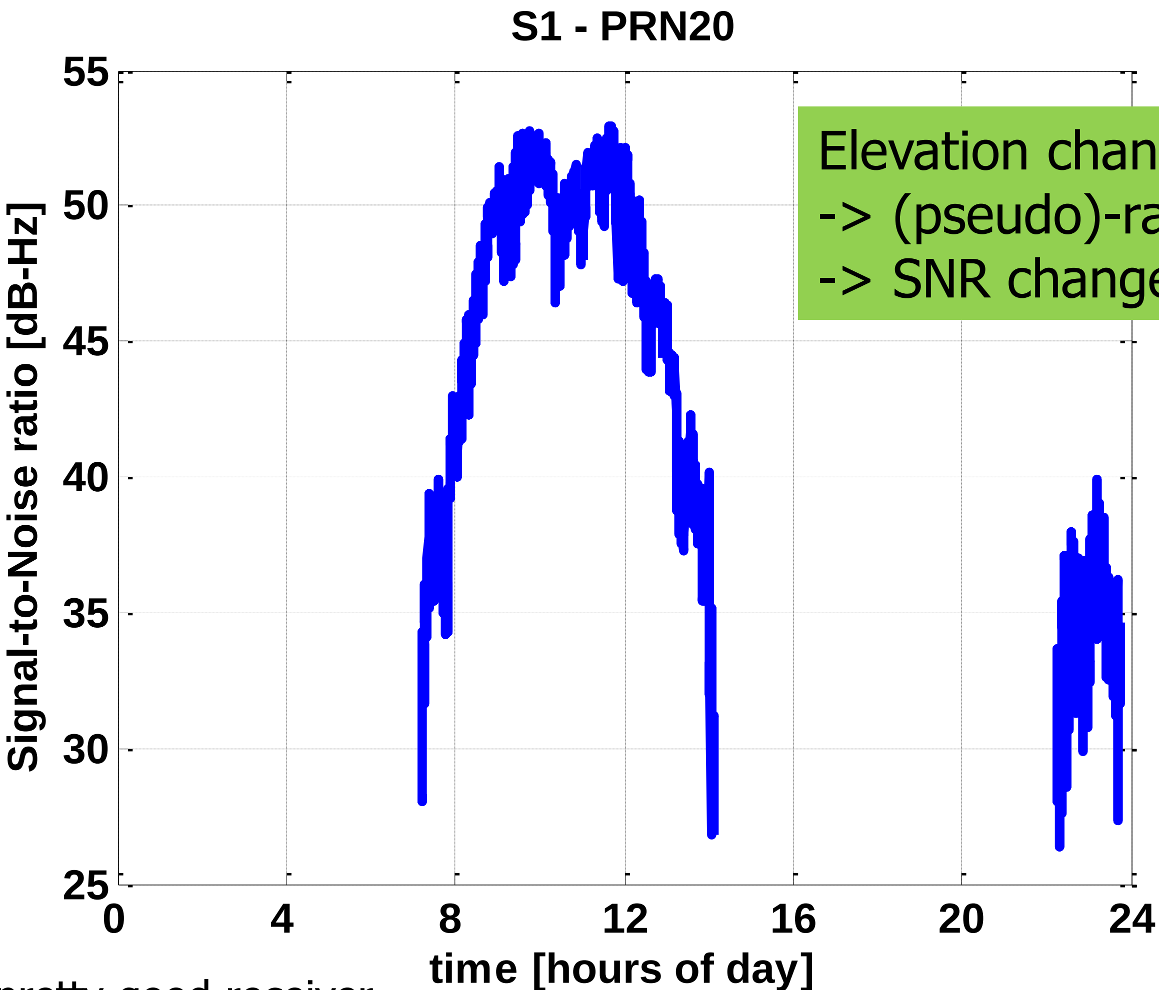
Pseudo-range observation



Why?

Signal strength

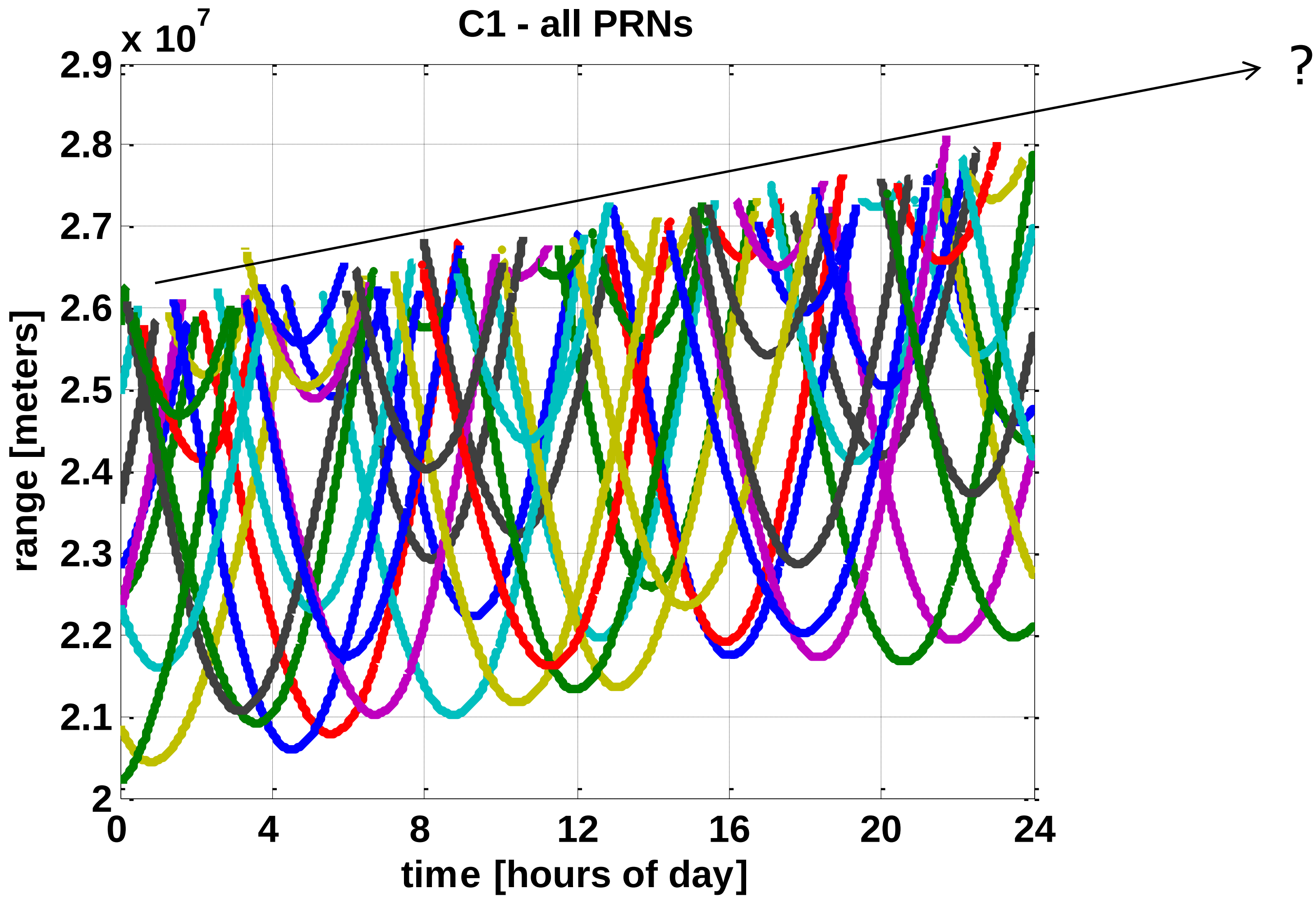
actually Carrier-to-Noise density ratio



Elevation changes, then
-> (pseudo)-range changes
-> SNR changes

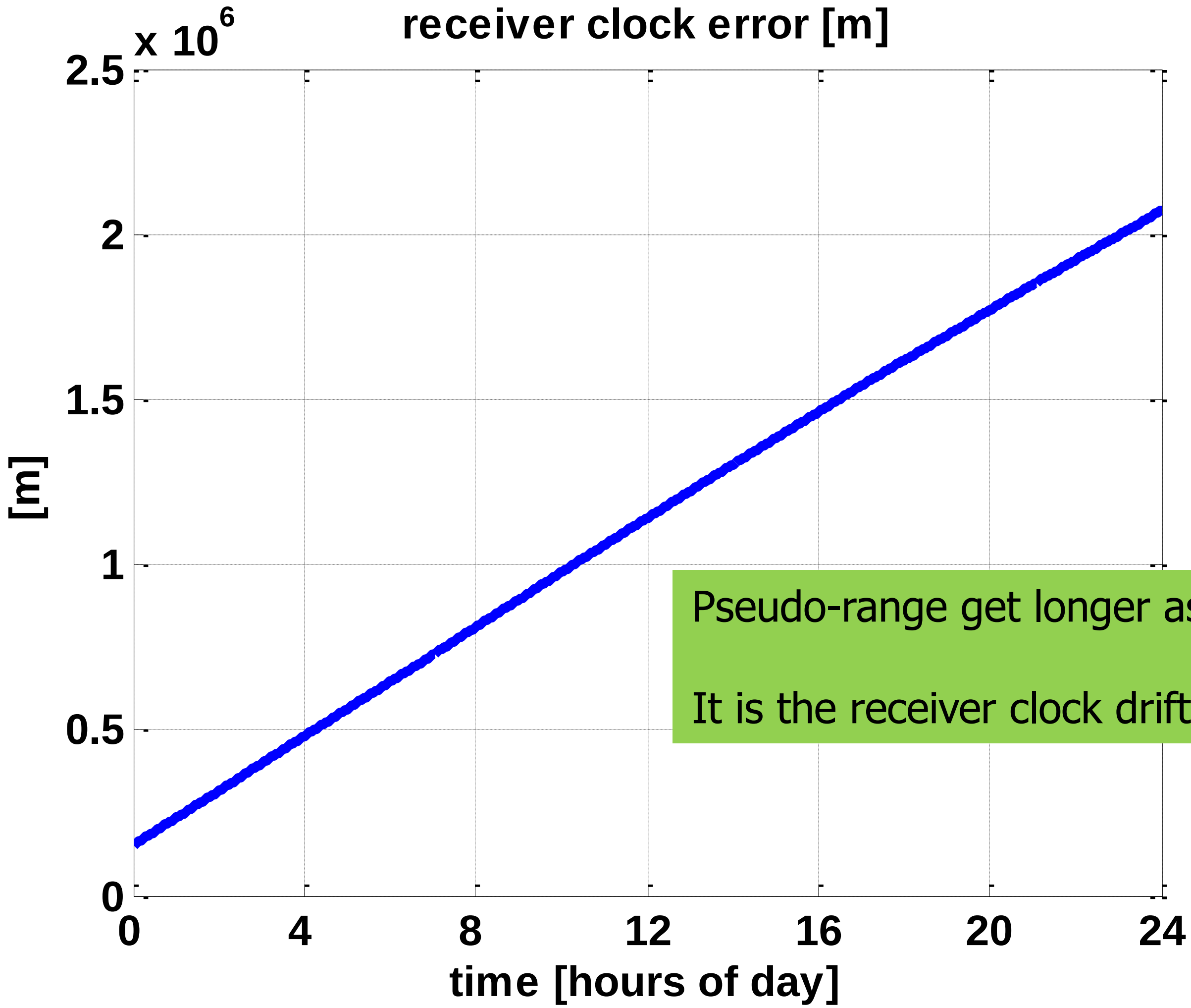
down to 25 dB-Hz, pretty good receiver …

Pseudo-range observations



they get longer as time proceeds …?

Receiver clock error



Pseudo-range get longer as time proceeds …?

It is the receiver clock drift!

oscillator in receiver has stability of about 10^{-7} s/s

Carrier phase measurements are more precise than pseudo-range measurements (dm → mm)

Fundamentals of GNSS Processing

CARRIER PHASE MEASUREMENT

L1 wavelength about 19cm

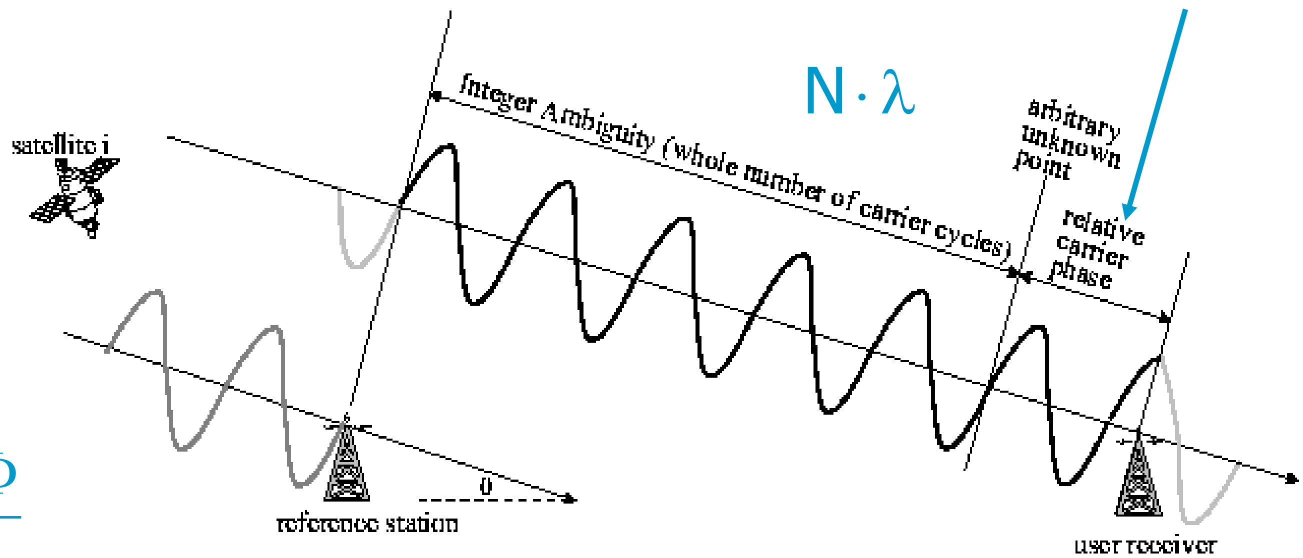
Carrier Phase Measurements

Use carrier phase Φ instead of pseudo-ranges

- phase measurement accuracy 1% of $\lambda \Rightarrow$ **mm accuracy**
- have to determine ("fix") ambiguity N
    - Ambiguity resolution, or,
    - Code-Carrier phase smoothing

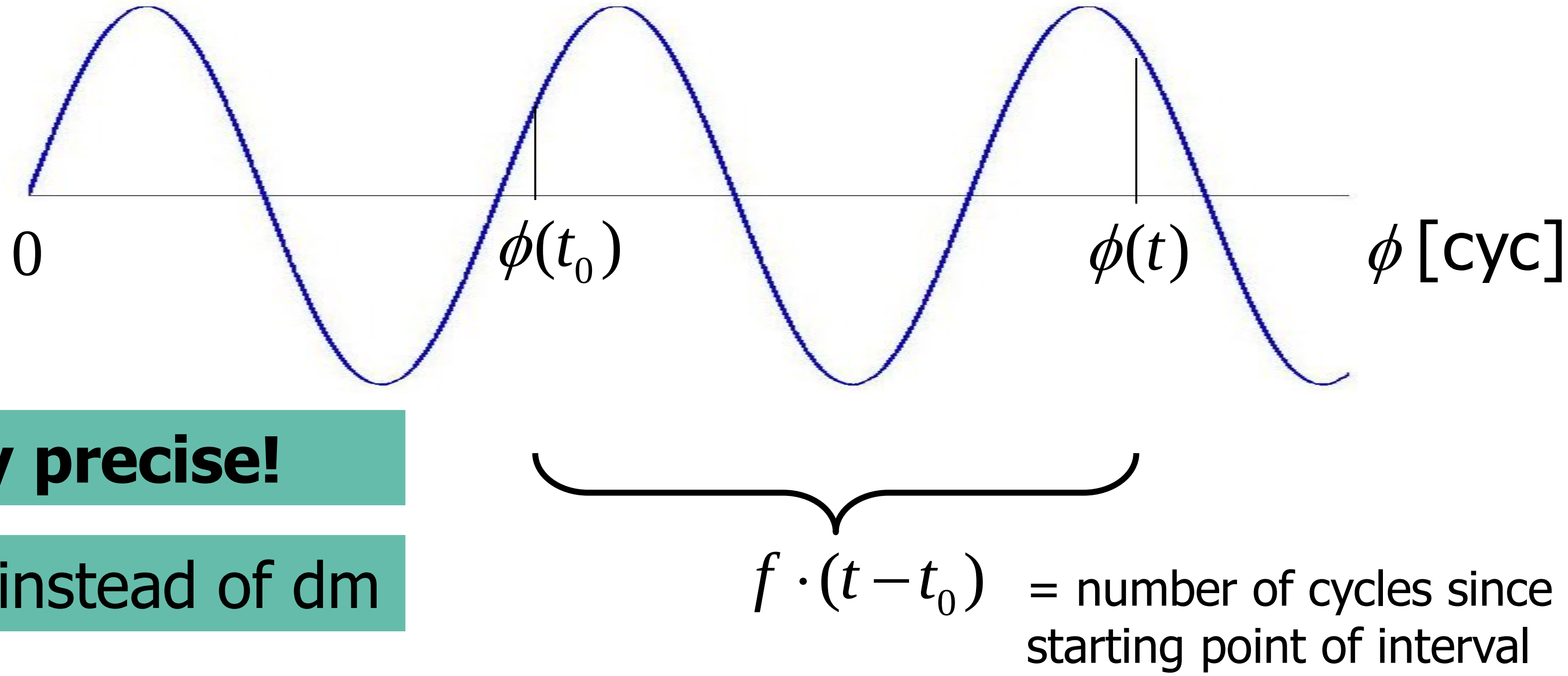
$$\frac{\lambda \cdot \Phi}{2\pi}$$

$$N \cdot \lambda$$



$$d = N \cdot \lambda + \frac{\lambda \cdot \Phi}{2\pi}$$

Carrier Phase measurements



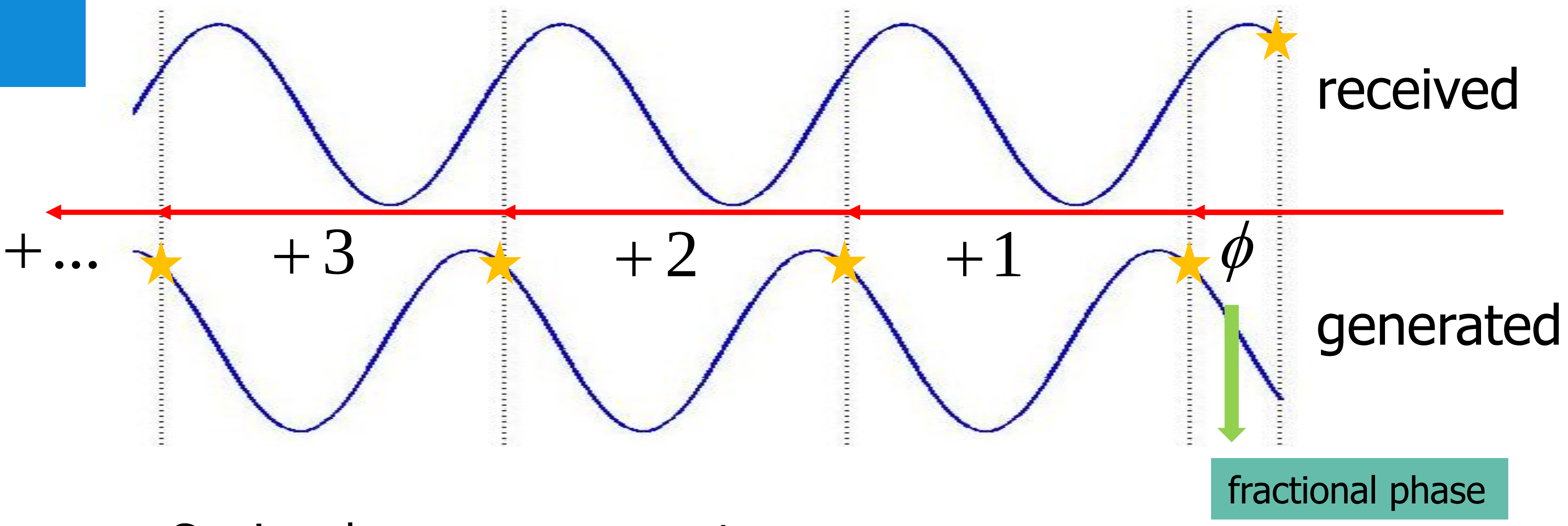
Very precise!

mm instead of dm

Carrier phase :

$$\phi(t) = \phi(t_0) + f \cdot (t - t_0)$$

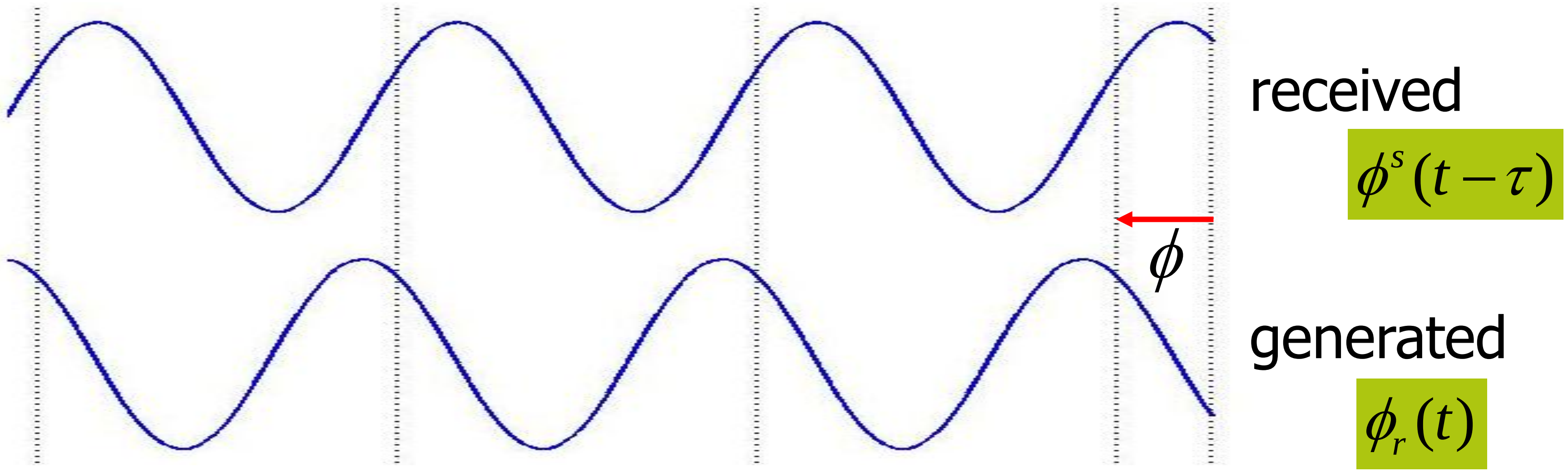
Carrier Phase measurements



Carrier phase measurement:

- Difference between phases of receiver-generated carrier signal and received carrier signal

Carrier Phase measurements



Carrier phase measurement:

- Difference between phases of receiver-generated carrier signal and received carrier signal
- Phase measurement + whole number of cycles traveled → range
- Change in phase continuously measured (incl. full cycles)

Carrier Phase measurements

$$\phi(t) = \phi_r(t) - \phi^s(t-\tau) + N + \varepsilon_\phi(t)$$

with

Recall: $\phi(t) = \phi(t_0) + f \cdot (t - t_0)$

$$\phi_r(t) = \phi_r(t_0) + f \cdot (t - t_0) + f \cdot (\delta t_r(t) - \delta t_r(t_0))$$

clock biases

$$\phi^s(t-\tau) = \phi^s(t_0) + f \cdot (t - \tau - t_0) + f \cdot (\delta t^s(t-\tau) - \delta t^s(t_0))$$

Compare this with the pseudo range observation – very similar derivation

Carrier Phase measurements

$$\phi(t) = \phi_r(t) - \phi^s(t-\tau) + N + \varepsilon_\phi(t)$$

$$\phi_r(t) = \phi_r(t_0) + \cancel{f \cdot (t - t_0)} + f \cdot \left(\delta t_r(t) - \delta t_r(t_0)\right)$$

$$\phi^s(t-\tau) = \phi^s(t_0) + f \cdot (\cancel{t} - \tau - \cancel{t_0}) + f \cdot \left(\delta t^s(t-\tau) - \delta t^s(t_0)\right)$$

$$\phi(t) = f \cdot \tau$$

$$+ f \cdot \left(\delta t_r(t) - \delta t^s(t-\tau)\right)$$ - clock biases

$$+\phi_r(t_0) - \phi^s(t_0)$$ - initial phases

$$- f \cdot \left(\delta t_r(t_0) - \delta t^s(t_0)\right)$$ - clock biases at t_0

$$+N$$ - integer ambiguity (constant)

(initial phases, clock biases at t_0 and integer ambiguity together: A)

$$+\varepsilon_\phi(t)$$ - noise and errors

Carrier Phase measurements

$$f = \frac{c}{\lambda}$$

$$\begin{aligned}\phi(t) &= f \cdot \tau \\ &+ f \cdot \left(\delta t_r(t) - \delta t^s(t-\tau)\right) \\ &+ A \\ &+ \varepsilon_\phi(t)\end{aligned}$$

cycles

$$\begin{aligned}\lambda \cdot \phi(t) &= c \cdot \tau \\ &+ c \cdot \left(\delta t_r(t) - \delta t^s(t-\tau)\right) \\ &+ \lambda \cdot A \\ &+ \lambda \cdot \varepsilon_\phi(t)\end{aligned}$$

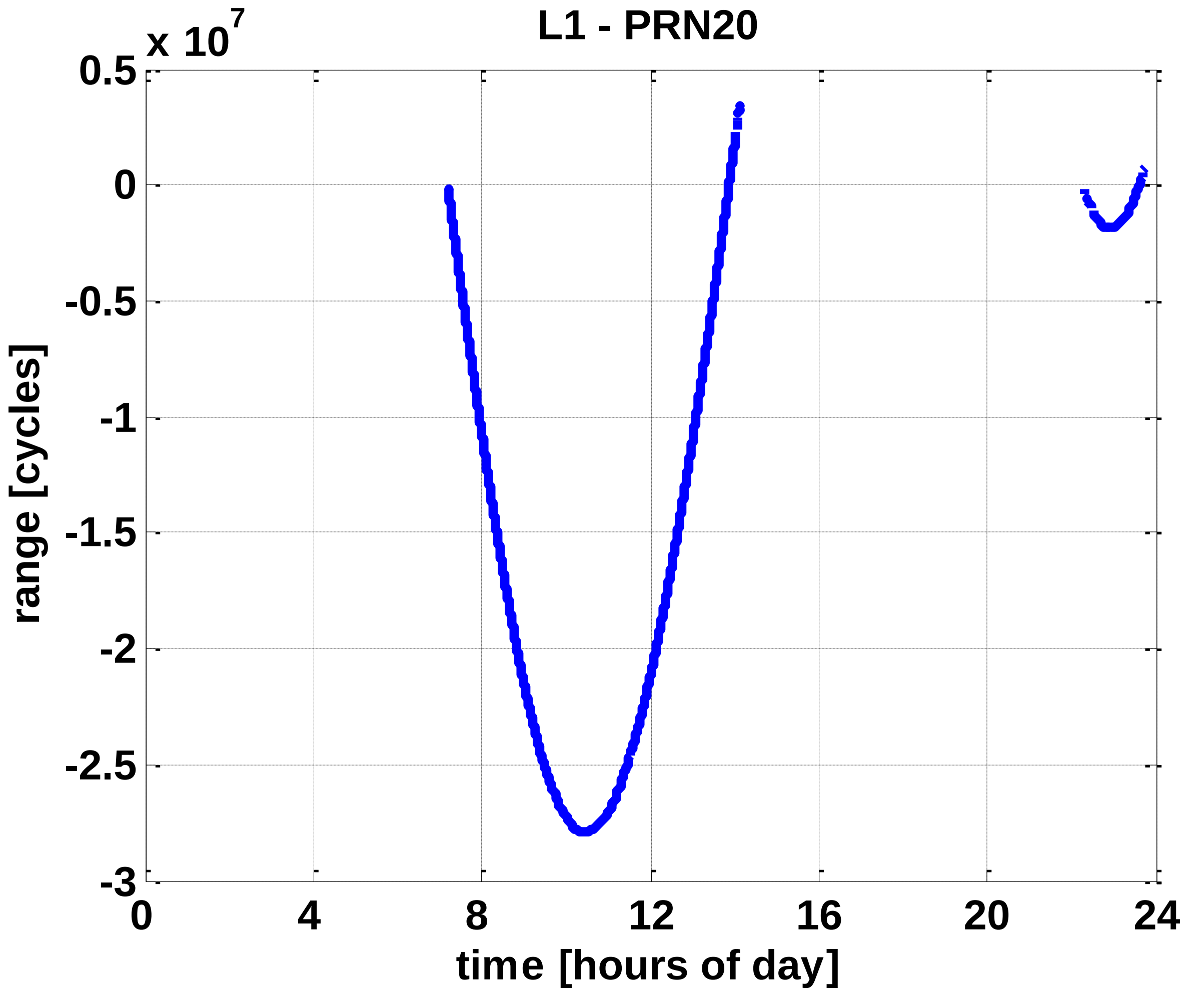
meters

$$\begin{aligned}\Phi &= r - \frac{f_1^2}{f_i^2} I_{L1} + T \\ &+ c \cdot \left(\delta t_r - \delta t^s\right) \\ &+ \lambda \cdot A \\ &+ \varepsilon_\Phi\end{aligned}$$

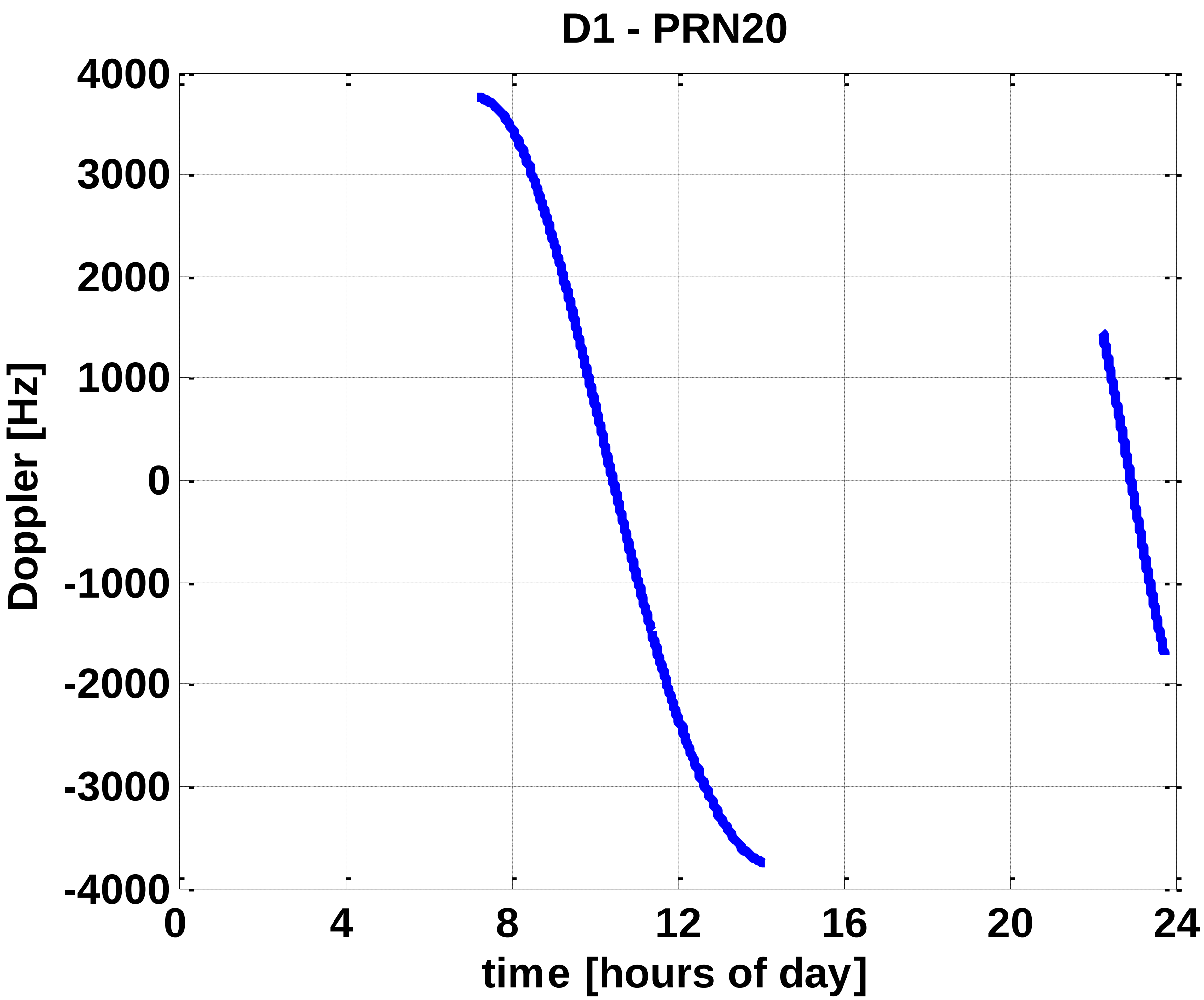
$$c \cdot \tau = r - \frac{f_1^2}{f_i^2} I_{L1} + T$$

L1 carrier phase observation

carrier phase is ambiguous (just starts at zero here)



L1 Doppler frequency observation



Code and Carrier Phase Equation

- side by side-

Constant in time!

Ionosphere is dispersive

$$\rho_{Li} = r + \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_r - \delta t^s\right] + \varepsilon_{\rho_{Li}}$$

$$\Phi_{Li} = r - \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_r - \delta t^s\right] + \lambda_{Li} A_{Li} + \varepsilon_{\Phi_{Li}}$$

$$\sigma_\rho \approx 20-100 \text{ cm} \ , \quad \sigma_\phi \approx 1-2 \text{ mm}$$

Note: All parameters depend on t, except for ambiguities!

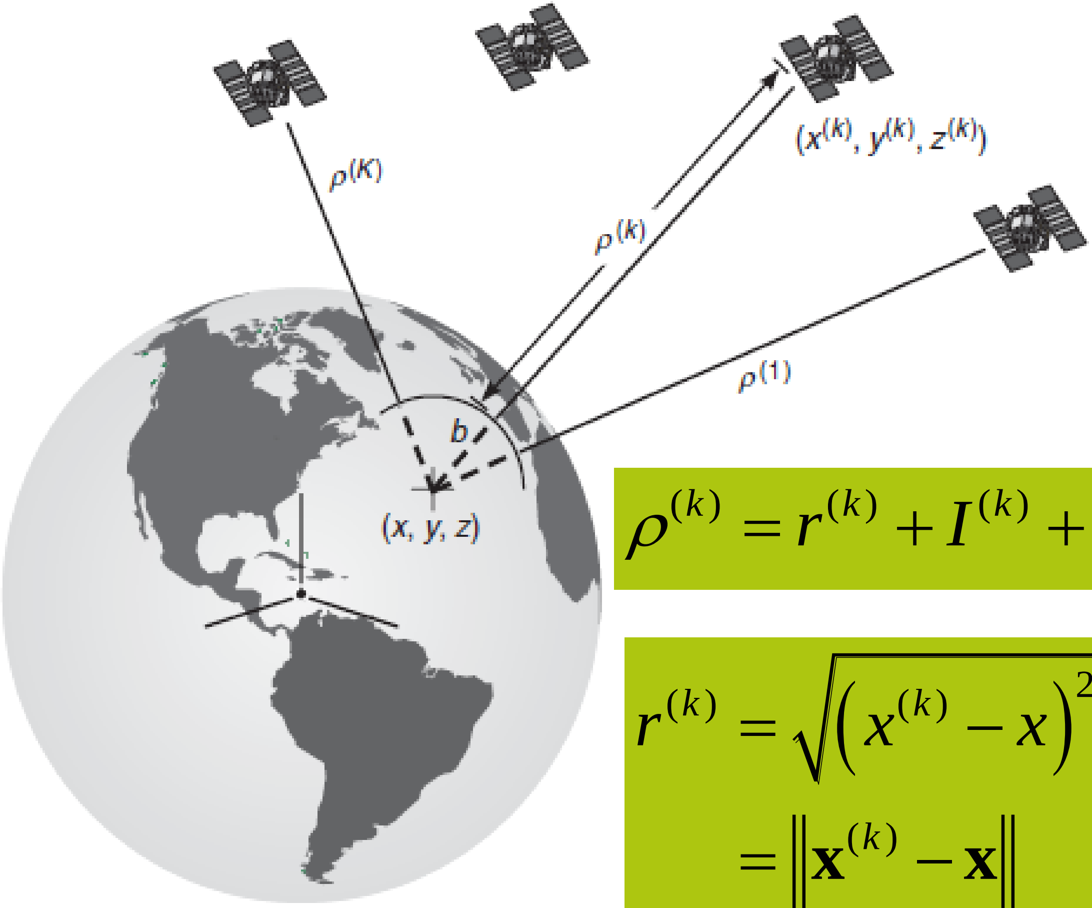
Note: Ambiguities must be resolved to take advantage of high precision phase measurements how to determine the GNSS position

Fundamentals of GNSS Processing

STANDARD GNSS NAVIGATION SOLUTION

GNSS Navigation

Non-linear code observation equations



$$\overset{\boldsymbol{b}\,\uparrow}{}$$

$$\rho^{(k)} = r^{(k)} + I^{(k)} + T^{(k)} + c\left[\delta t_r - \delta t^{(k)}\right] + \varepsilon_\rho^{(k)}$$

$$r^{(k)} = \sqrt{\left(x^{(k)} - x\right)^2 + \left(y^{(k)} - y\right)^2 + \left(z^{(k)} - z\right)^2}$$
$$= \left\| \mathbf{x}^{(k)} - \mathbf{x} \right\|$$

Linearization Basics

non-linear model $\mathbf{y} = \mathbf{H}(\mathbf{v}) + \boldsymbol{\varepsilon}$

Taylor series $\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}\left(\mathbf{v} - \mathbf{v}_0\right) + \ldots + \boldsymbol{\varepsilon}$

observed-minus-computed observations $\delta\mathbf{y} = \mathbf{y} - \mathbf{H}(\mathbf{v}_0)$

correction to approximate values $\delta\mathbf{v} = \mathbf{v} - \mathbf{v}_0$

design matrix $\mathbf{A} = \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}$

linearized model:

$$\delta\mathbf{y} = \mathbf{A}\delta\mathbf{v} + \boldsymbol{\varepsilon}$$

GNSS Linearization…

$$\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}\left(\mathbf{v} - \mathbf{v}_0\right) + \ldots + \boldsymbol{\varepsilon}$$

Observed minus computed:

b_o

$$\rho_0^{(k)} = \left\| \mathbf{x}^{(k)}{}_0 - \mathbf{x}_0 \right\| + c\delta t_{r0} - c\delta t_0^{(k)} + I^{(k)}{}_0 + T^{(k)}{}_0$$

$$\mathbf{H}(\mathbf{v}_0)$$

$$\delta\rho^{(k)} = \rho^{(k)} - \rho_0^{(k)}$$

$$\mathbf{y} - \mathbf{H}(\mathbf{v}_0)$$

Approximations required for:

- **satellite position** at time of transmission t - τ (from ephemeris) problem: τ not precisely known (satellite moves ~300m in signal transit time)
- **atmospheric delays**
- **receiver position** at time of reception t
- **receiver clock error**

} Initial values for iteration

GNSS Linearization…

$$\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}\left(\mathbf{v} - \mathbf{v}_0\right) + \ldots + \boldsymbol{\varepsilon}$$

Design matrix:

$$\mathbf{A} = \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}$$

Partial derivatives:

- Coordinate part

$$\begin{bmatrix} \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial x} \\ \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial y} \\ \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial z} \end{bmatrix}^T = \begin{bmatrix} -\frac{x^{(k)}{}_0 - x_0}{\left\| \mathbf{x}^{(k)}{}_0 - \mathbf{x}_0 \right\|} \\ -\frac{y^{(k)}{}_0 - y_0}{\left\| \mathbf{x}^{(k)}{}_0 - \mathbf{x}_0 \right\|} \\ -\frac{z^{(k)}{}_0 - z_0}{\left\| \mathbf{x}^{(k)}{}_0 - \mathbf{x}_0 \right\|} \end{bmatrix}^T = (-\mathbf{1}^{(k)})^T$$

- Receiver clock part

$$\frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial b} = 1$$

Unit direction vector (receiver to satellite)

GNSS Linearization…

$$\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}(\mathbf{v} - \mathbf{v}_0) + \ldots + \boldsymbol{\varepsilon}$$

Linearized code observation equations:

Single satellite:

$$\delta\rho^{(k)} = (-\mathbf{1}^{(k)})^T \delta\mathbf{x} + \delta b + \tilde{\varepsilon}_\rho^{(k)}$$

$$\delta\mathbf{y} = \mathbf{A}\delta\mathbf{v} + \boldsymbol{\varepsilon}$$

All satellites in view:

$$\boldsymbol{\delta\rho} = \begin{bmatrix} \delta\rho^{(1)} \\ \delta\rho^{(2)} \\ \vdots \\ \delta\rho^{(m)} \end{bmatrix} = \begin{bmatrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \delta b \end{bmatrix} + \tilde{\boldsymbol{\varepsilon}}_\rho$$

$m \times 1$ $\qquad \underbrace{\qquad}_{\boldsymbol{G}}$ $m \times 4$ $\qquad 4 \times 1$

$$\delta\mathbf{v} = \mathbf{v} - \mathbf{v}_0$$

$$\delta\mathbf{x} = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}$$

$$\delta b = b - b_0$$

Least-squares Solution (BLUE)

Linear model:

$$\partial \mathbf{y} = \mathbf{A}\partial \mathbf{v} + \boldsymbol{\varepsilon}; \qquad \mathbf{Q}_{\mathbf{yy}}$$

variance matrix observations

Least squares solution:

$$\mathbf{W} = \mathbf{Q}_{\mathbf{yy}}^{-1}$$

Weight matrix is the inverse covariance matrix

$$\partial \hat{\mathbf{x}} = \underbrace{\left(\mathbf{A}^{T}\mathbf{Q}_{\mathbf{yy}}^{-1}\mathbf{A}\right)^{-1}}_{\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}}} \mathbf{A}^{T}\mathbf{Q}_{\mathbf{yy}}^{-1}\partial \mathbf{y}$$

variance matrix observations

Least-squares Solution (GNSS)

Linearized model:

$$\boldsymbol{\delta\rho} = \mathbf{G}\delta\mathbf{v} + \tilde{\boldsymbol{\varepsilon}}; \qquad \mathbf{Q}_{\rho\rho}$$

Least squares solution:

$$\begin{pmatrix} \delta\hat{\mathbf{x}} \\ \delta\hat{b} \end{pmatrix} = \underbrace{\left(G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}G\right)^{-1}}_{\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}}} G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\delta\boldsymbol{\rho}$$

Must be iterated until solution converges (Gauss-Newton method)

$\mathbf{v}_0 = \ldots$

while $\left\|\delta\hat{\mathbf{v}}\right\|^2_{Q_{\hat{v}\hat{v}}} \geq \eta$

$\quad \delta\boldsymbol{\rho} = \boldsymbol{\rho} - \mathbf{H}(\mathbf{v}_0)$

$\quad \mathbf{G} = \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}$

$\quad Q_{\hat{v}\hat{v}} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\mathbf{G}\right)^{-1}$

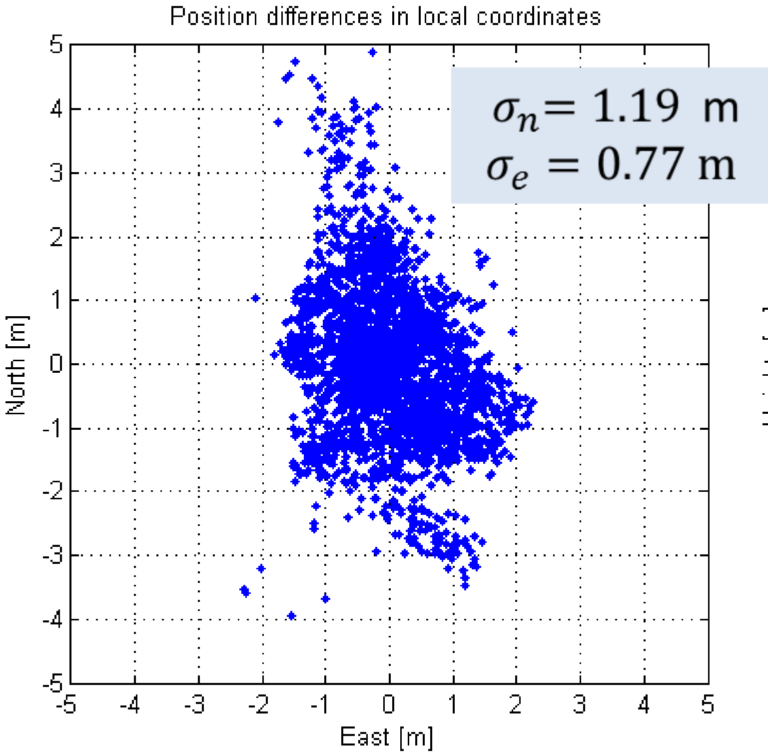
$\quad \delta\hat{\mathbf{v}} = Q_{\hat{v}\hat{v}} \cdot \mathbf{G}^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\delta\boldsymbol{\rho}$

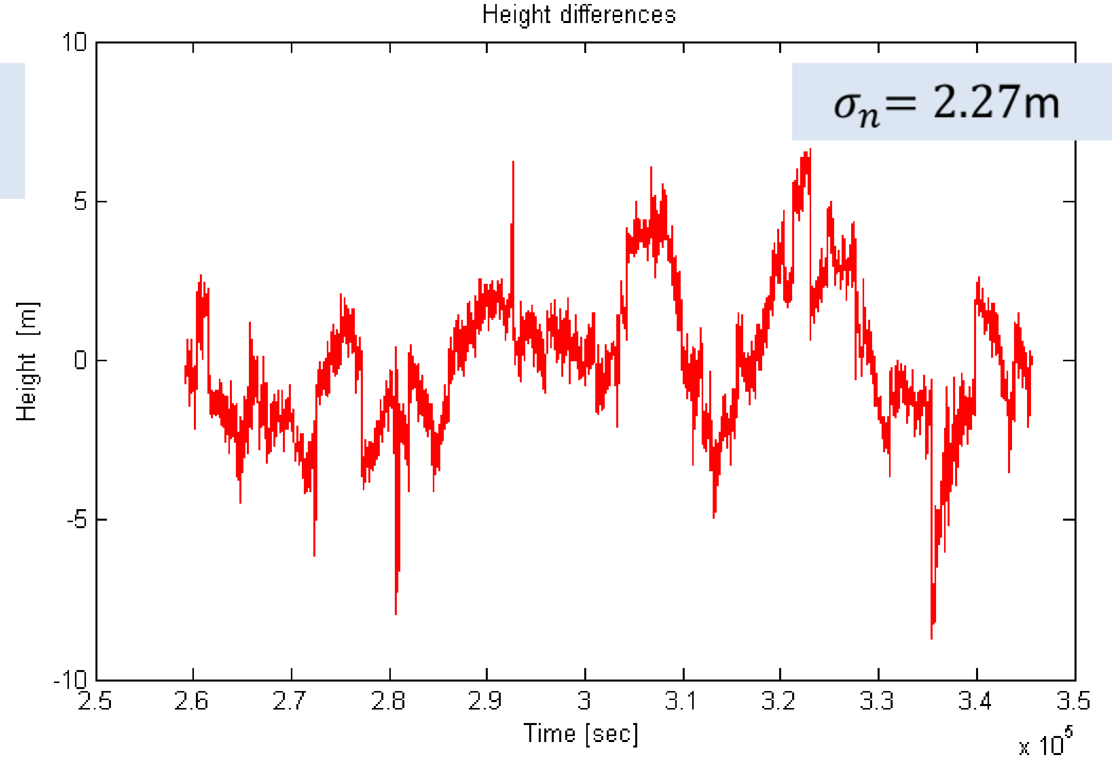
$\quad \hat{\mathbf{v}} = \mathbf{v}_0 + \delta\hat{\mathbf{v}}$

$\quad \mathbf{v}_0 = \hat{\mathbf{v}}$

end

Must be iterated until the solution converges…



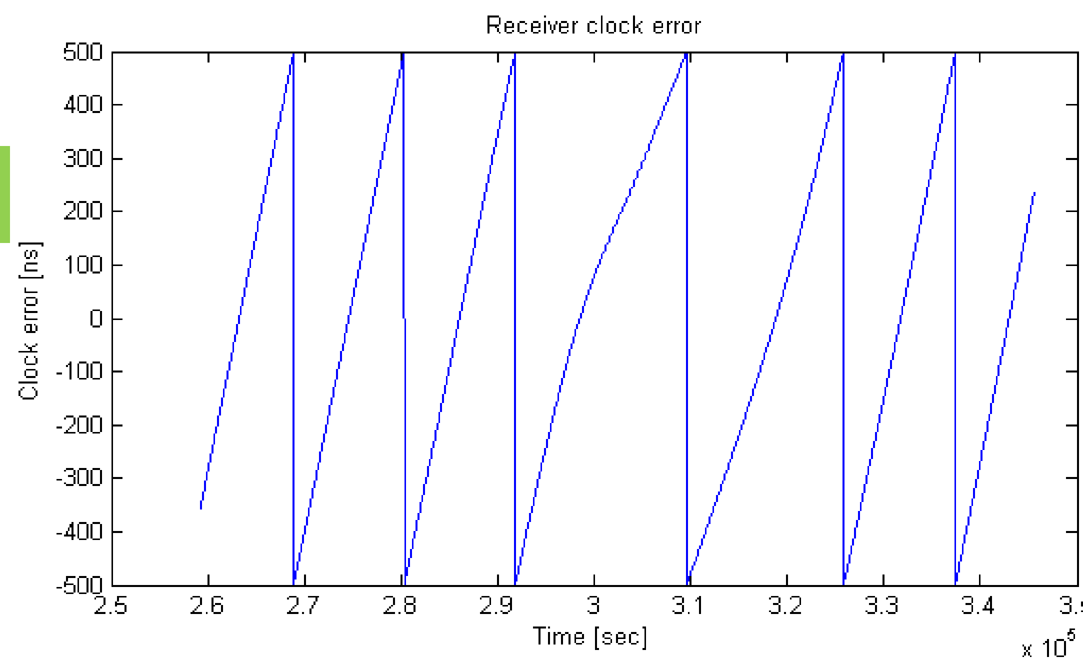


Trimble R7 Cabauw

- 24 March 2010
- L1 C/A code
- 24 hours

example

Standard navigation solution



COV

Precision of the LSQ Solution

Least-squares solution:

$$\begin{pmatrix} \delta\hat{\mathbf{x}} \\ \delta\hat{b} \end{pmatrix} = \underbrace{\left(G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}G\right)^{-1}}_{\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}}} G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\delta\boldsymbol{\rho}$$

Depends on:
- Observations $\delta\boldsymbol{\rho}$
- Satellite geometry G
- Covariance matrix observations $\mathbf{Q}_{\delta\rho\delta\rho}$

Covariance matrix of the LSQ Solution:

$$\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}} = \left(G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}G\right)^{-1}$$

Depends on:
- Satellite geometry G
- Covariance matrix observations $\mathbf{Q}_{\delta\rho\delta\rho}$

Covariance matrix of LSQ sol. (=precision) does not depend on the actual observations…

Precision of the GNSS solution

Precision depends on

1) Co-variance matrix of the linearized observations
   - Pseudo-range measurement noise } $\mathbf{Q}_{\rho\rho}$
   - Satellite ephemeris and clock errors
   - Atmospheric delays
   - Multipath effects

   (all of the above) } $\mathbf{Q}_{\delta\rho\delta\rho}$

COV

$\mathbf{Q}_{\delta\rho\delta\rho} \neq \mathbf{Q}_{\rho\rho}$

URE

2) Satellite Geometry (Dillution of Precision = DOP)

DOP

Integrity of the computed positions and precision measures is provided by e.g. overall-model test

OMT

Covariance Matrix of the Observations

Observed minus computed:

URE

$$\delta\rho^{(k)} = \rho^{(k)} - \rho_0^{(k)}$$

Variance of the observations:

$$D\{\delta\rho^{(k)}\} = D\{\rho^{(k)}\} + \mathbf{1}^{(k)T} D\{\mathbf{x}_0{}^{(k)}\}\mathbf{1}^{(k)} + D\{c\delta t_0^{(k)}\} + D\{I^{(k)}{}_0\} + D\{T^{(k)}{}_0\}$$

Includes:
- Pseudo range noise and multipath
- Satellite orbit and clock errors
- Atmospheric errors

$\Rightarrow$ $\mathbf{Q}_{\delta\rho\delta\rho} \neq \mathbf{Q}_{\rho\rho}$

GPS Error Budget

Empirical values, actual values depend on receiver, atmosphere models, time and location

Error source	RMS range error [m]
satellite clock and ephemeris	$\sigma_{RE/CS} = 3$ m = SIS URE
atmospheric propagation modeling	$\sigma_{RE/P} = 5$ m
receiver noise and multipath	$\sigma_{RE/RNM} = 1$ m
User Range Error (URE)	$\sigma_{URE} = 6$ m

$$\sigma_{URE} = \sqrt{\sigma_{RE/CS}^2 + \sigma_{RE/P}^2 + \sigma_{RE/RNM}^2}$$

DOP

Satellite geometry

If: $\mathbf{W}^{-1} = \mathbf{Q}_{\rho\rho} = \sigma^2 I_K$ → $\mathbf{Q}_{\hat{v}\hat{v}} = \sigma^2 \underbrace{(G^T G)^{-1}}_{D} = \begin{pmatrix} \sigma_x^2 & & \text{cov} & \\ & \sigma_y^2 & & \\ & & \sigma_z^2 & \\ & \text{cov} & & \sigma_b^2 \end{pmatrix}$

RMS position error

$$\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} = \sigma \underbrace{\sqrt{D_{11} + D_{22} + D_{33}}} = \sigma \cdot \text{PDOP}$$

influence of satellite geometry on precision
→ Position Dilution of Precision (PDOP)

RMS clock bias error

$$\sqrt{\sigma_b^2} = \sigma \sqrt{D_{44}} = \sigma \cdot \text{TDOP}$$

RMS overall error

$$\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_b^2} = \sigma \sqrt{D_{11} + D_{22} + D_{33} + D_{44}} = \sigma \cdot \text{GDOP}$$

Overall Model test

OMT

Least-squares solution:

$$\begin{pmatrix} \delta\hat{\mathbf{x}} \\ \delta\hat{b} \end{pmatrix} = \underbrace{\left(G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}G\right)^{-1}}_{\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}}} G^{\mathrm{T}}\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\delta\boldsymbol{\rho}$$

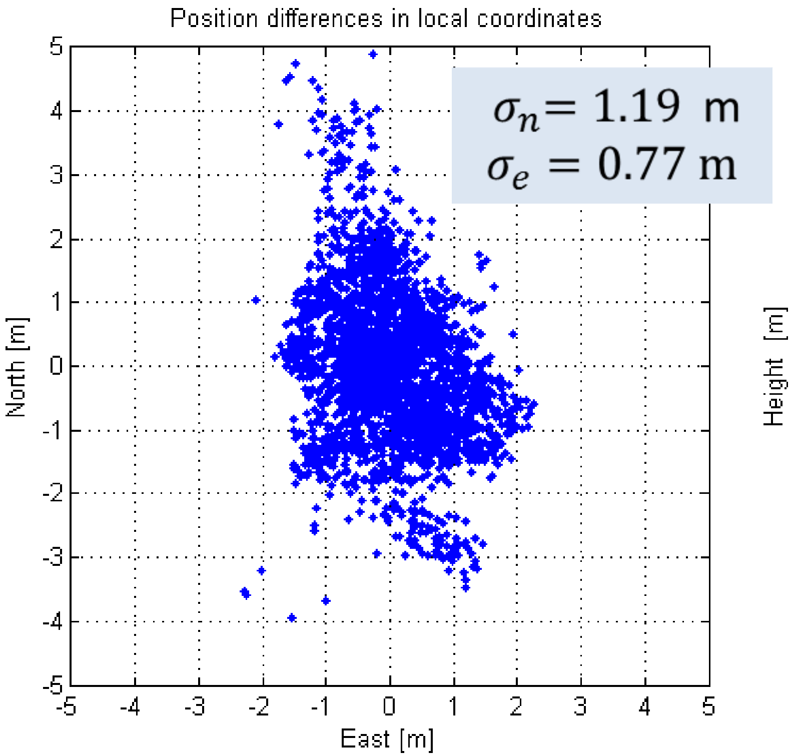
Least-squares residuals:

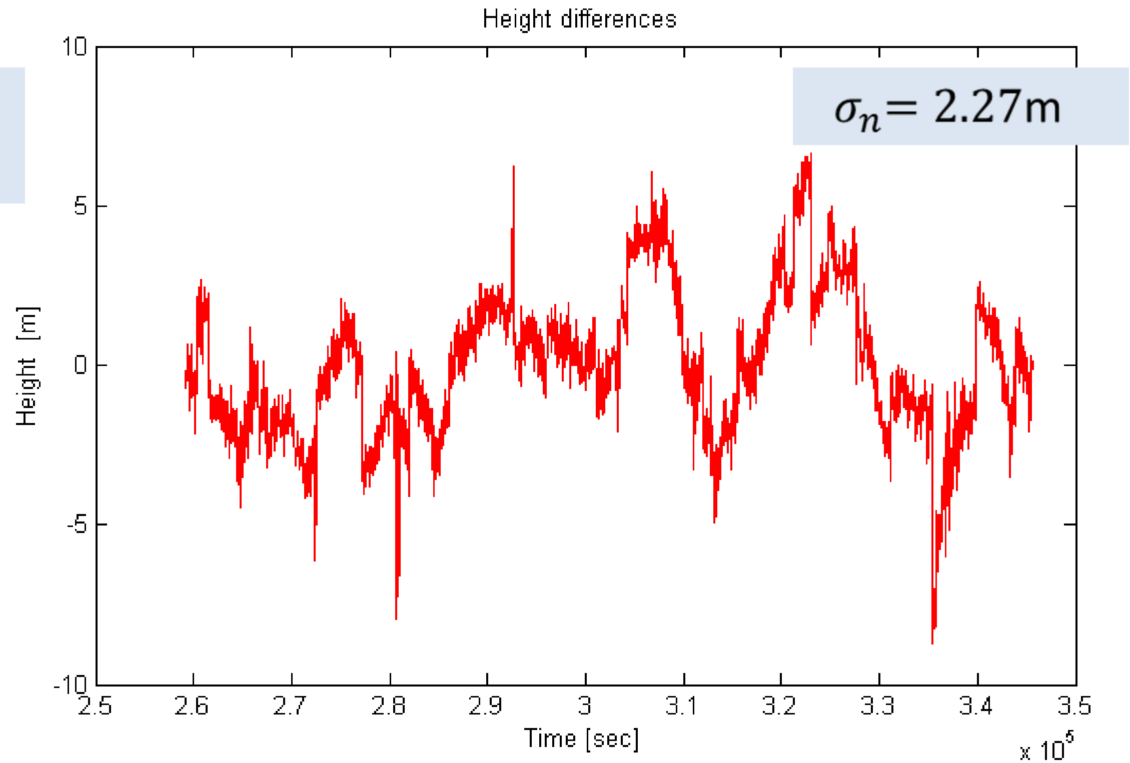
$$\mathbf{e} = \delta\boldsymbol{\rho} - G\begin{pmatrix} \delta\hat{\mathbf{x}} \\ \delta\hat{b} \end{pmatrix}$$

Overall model test:

$$T = \frac{\mathbf{e}^T\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\mathbf{e}}{m-4} \sim \chi^2(m-4)/(m-4) \rightarrow E\{T\}=1$$

Accept if $$T = \frac{\mathbf{e}^T\mathbf{Q}_{\delta\rho\delta\rho}^{-1}\mathbf{e}}{m-4} < \chi_k{}^2(\alpha; m-4)/(m-4)$$

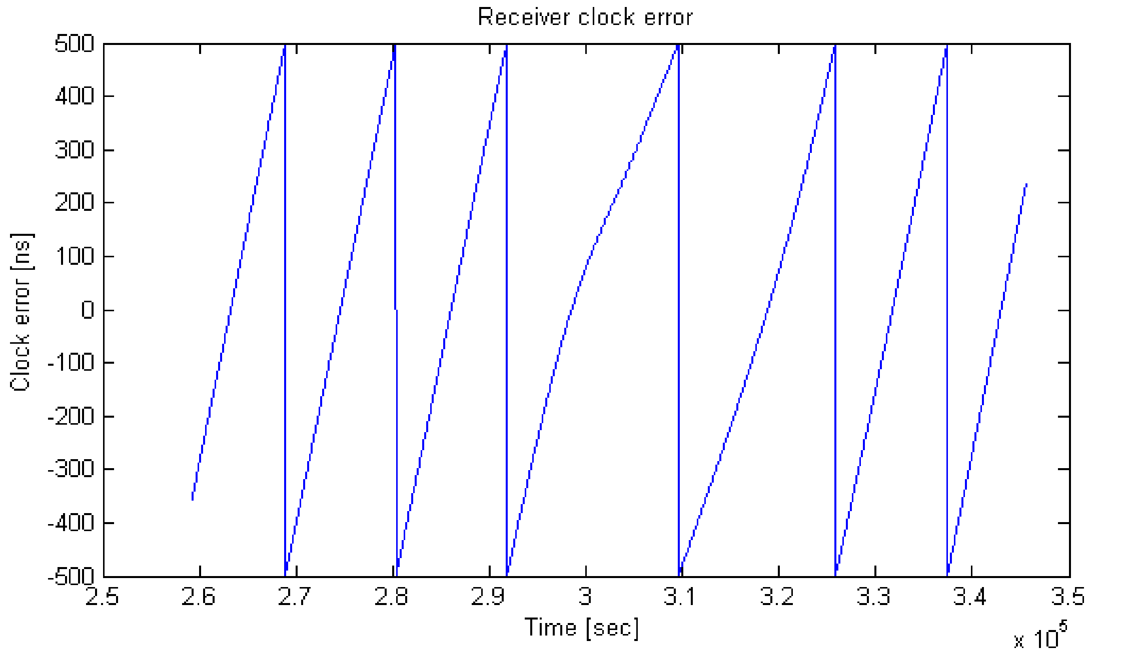




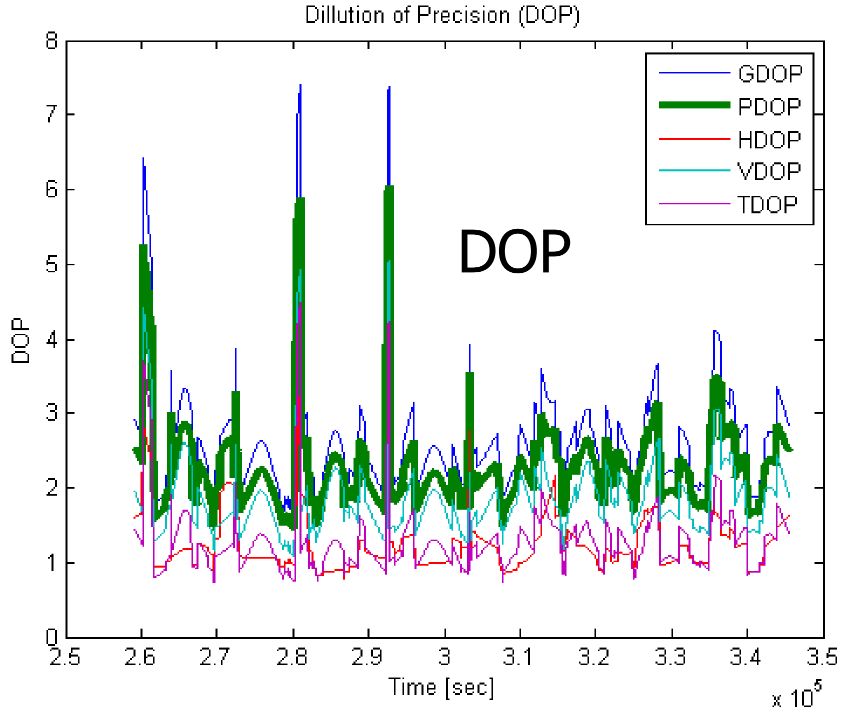
Trimble R7 Cabauw

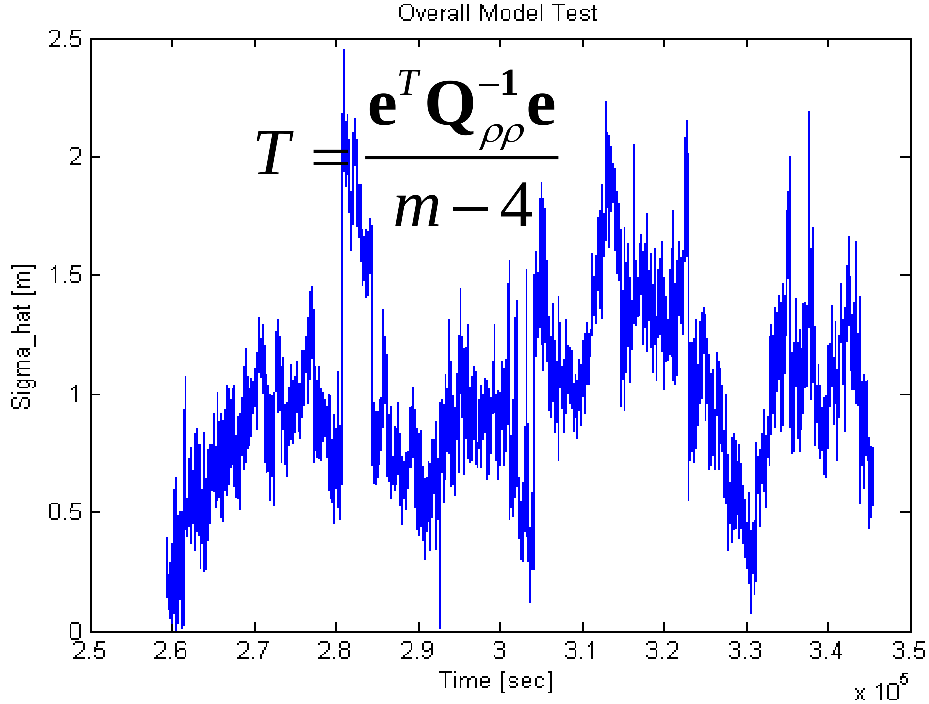
- 24 March 2010
- L1 C/A code
- 24 hours

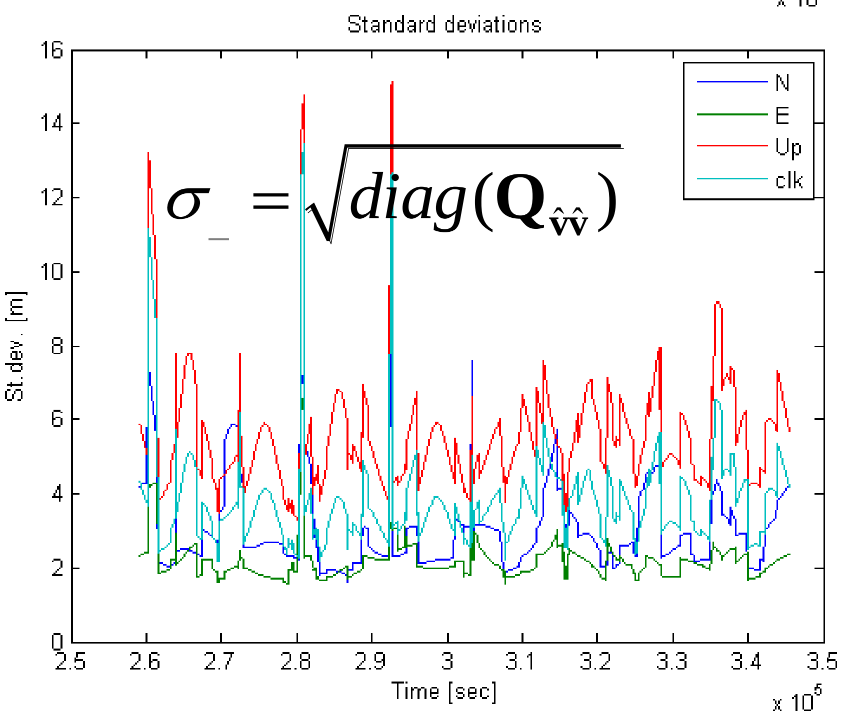
Standard navigation solution

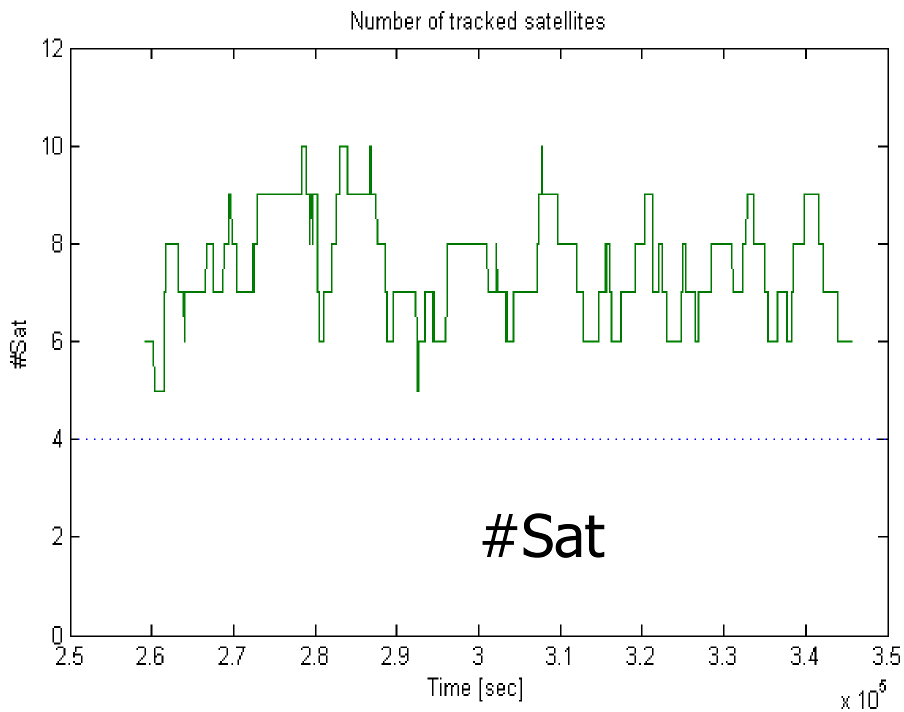


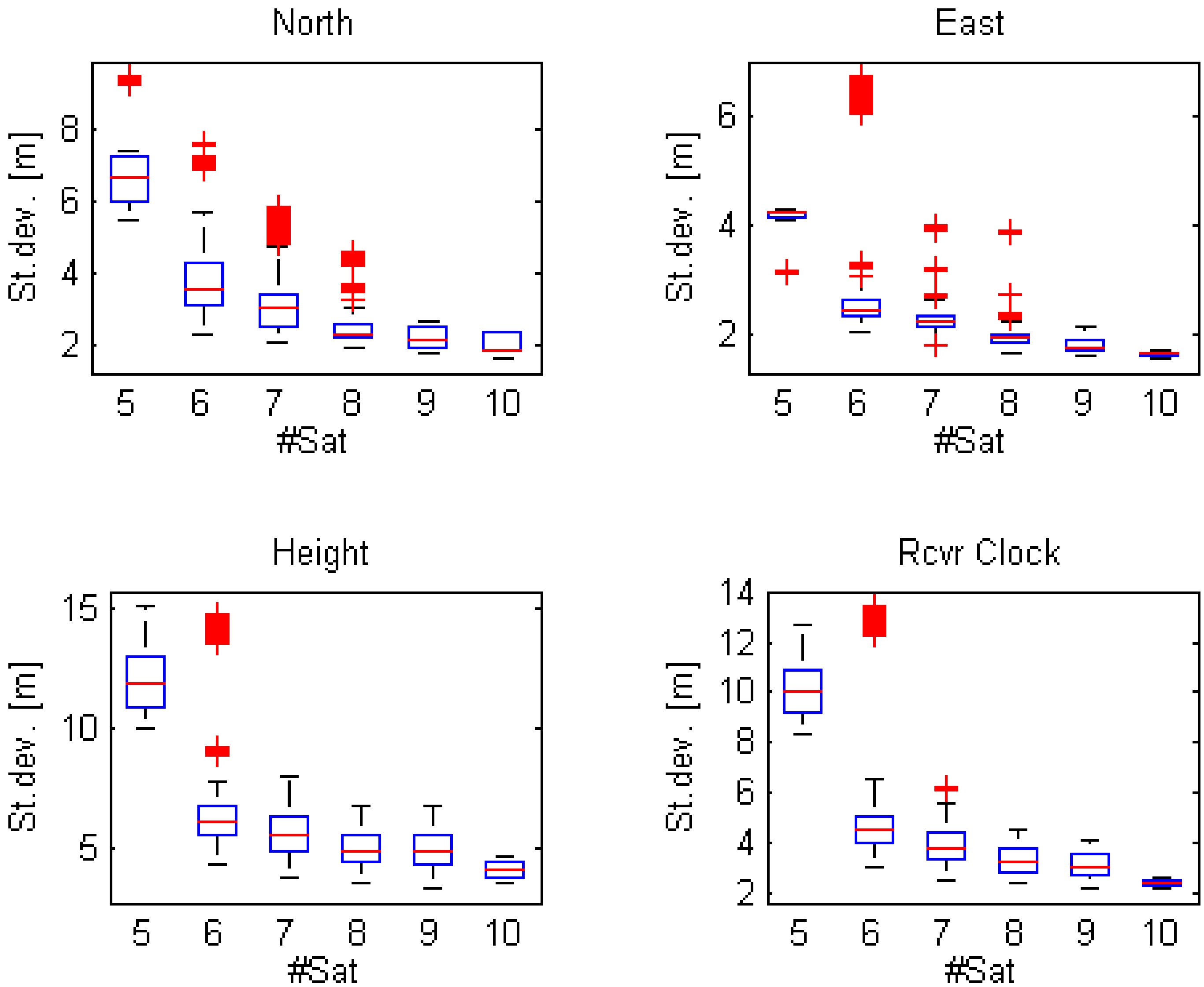
Example (cont')


Dillution of Precision (DOP)
GDOP
PDOP
HDOP
VDOP
TDOP
DOP
DOP
Time [sec]
x 10^5

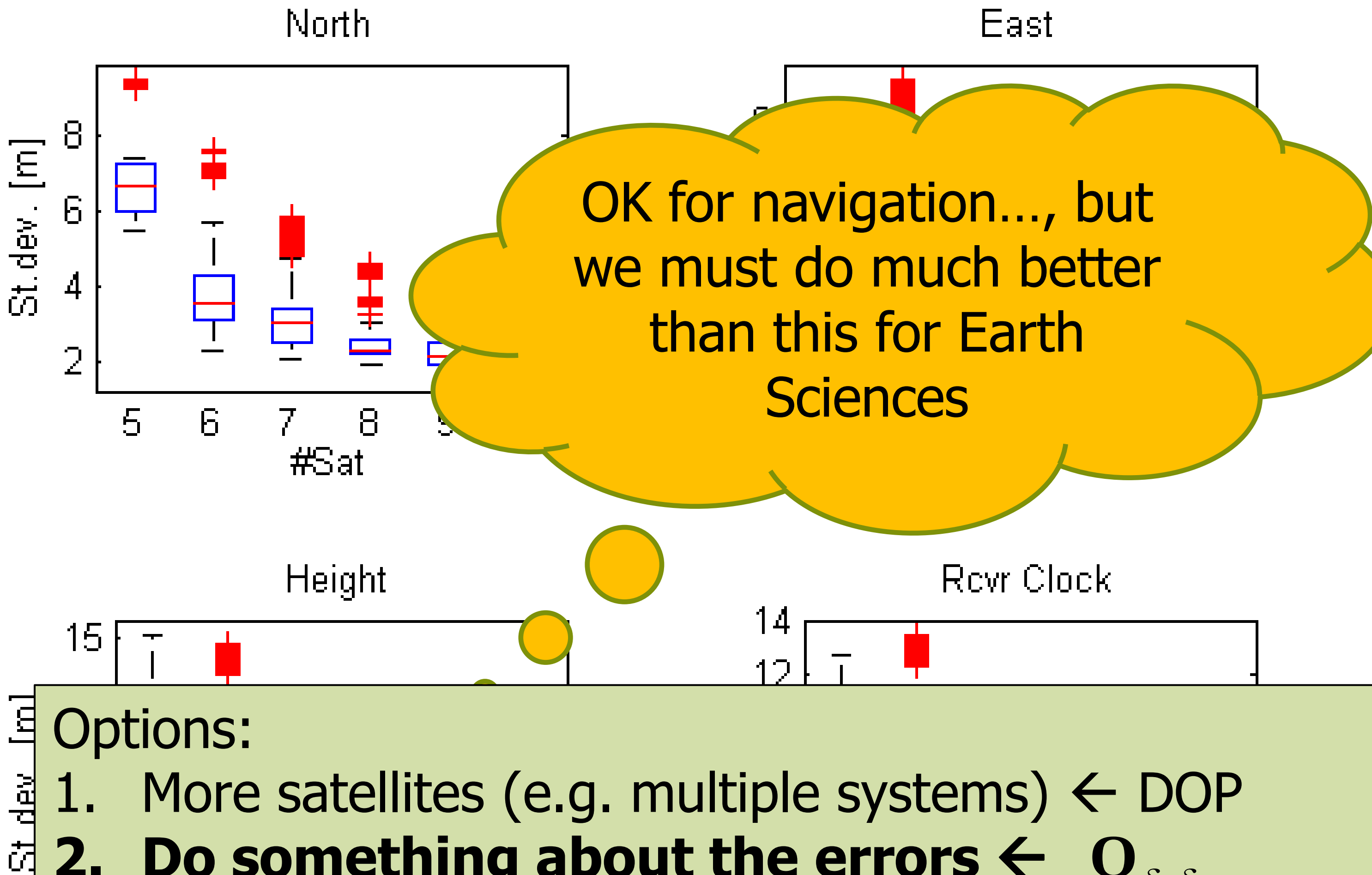

Overall Model Test
$T = \frac{\mathbf{e}^T \mathbf{Q}_{\rho\rho}^{-1} \mathbf{e}}{m-4}$
Sigma_hat [m]
Time [sec]
x 10^5


Standard deviations
N
E
Up
clk
$\sigma_{\_} = \sqrt{diag(\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}})}$
St.dev. [m]
Time [sec]
x 10^5


Number of tracked satellites
#Sat
#Sat
Time [sec]
x 10^5



Number of satellites in view has a big influence on the precision of the solution…



OK for navigation…, but we must do much better than this for Earth Sciences

Options:
1. More satellites (e.g. multiple systems) ← DOP
2. **Do something about the errors ← $\mathbf{Q}_{\delta\rho\delta\rho}$**

Let have a look at the error budget

Number of satellites in view has a big influence on the precision of the solution… but not enough

Which, how large, and what to do about it?

Fundamentals of GNSS Processing

GNSS ERROR SOURCES

GNSS error sources

- satellite
  - orbit
  - clock
  - instrumental delays *)
- signal path
  - ionosphere
  - troposphere
  - multipath
  - other (e.g. snow)
- receiver
  - Noise (code or phase)
  - instrumental delays *)
- antenna
  - phase center
  - elevation/azimuth dep.
- other
  - spoofing / interference
  - tides and loading
  - phase wind-up

*) between signals and systems

Satellite clock and ephemeris errors

- Broadcast ephemerides (orbits and clocks)
  - Determined by Control Segment based on measurements
  - Have been improved significantly over the last years
  - **Currently, ranging error < 2-3 meter rms**
  - **Together with ionosphere delay the largest error source**

- What can we do about it?

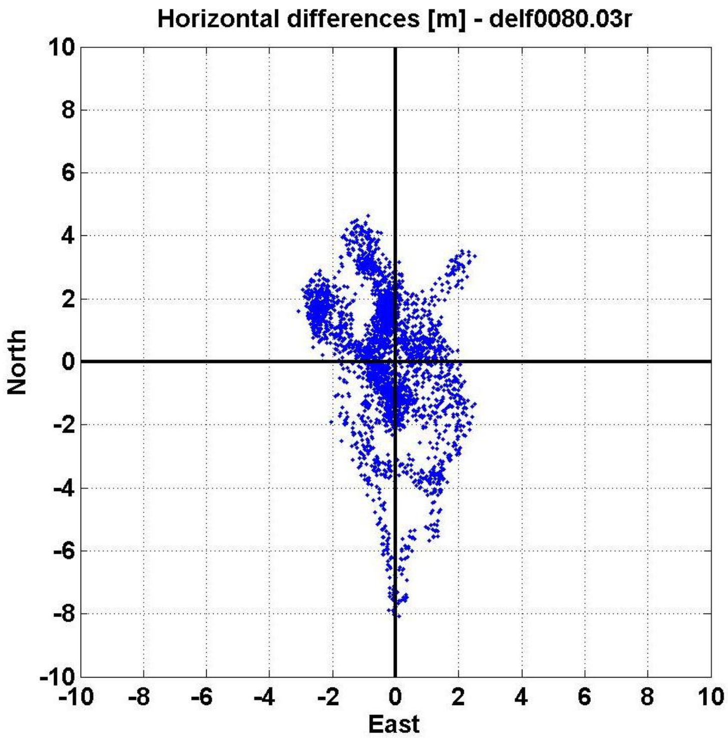
  -> Use differential GPS (**DGPS**) or relative positioning — Our choice

  -> Use precise orbits and clocks — Our choice

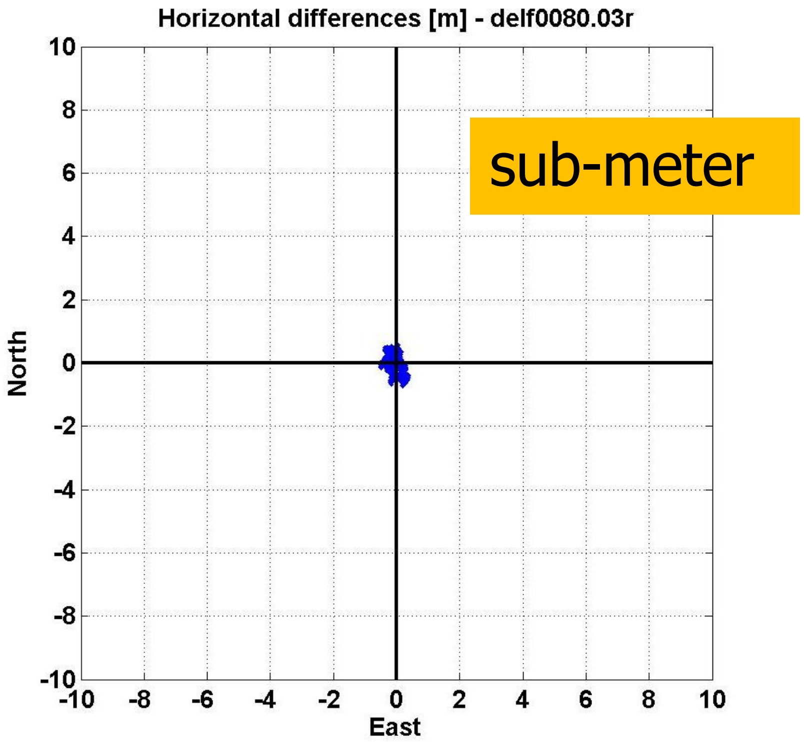
  - **International GNSS Service** (free, **cm level**)
  - Commercial Providers

Example of using IGS orbits&clocks

Broadcast



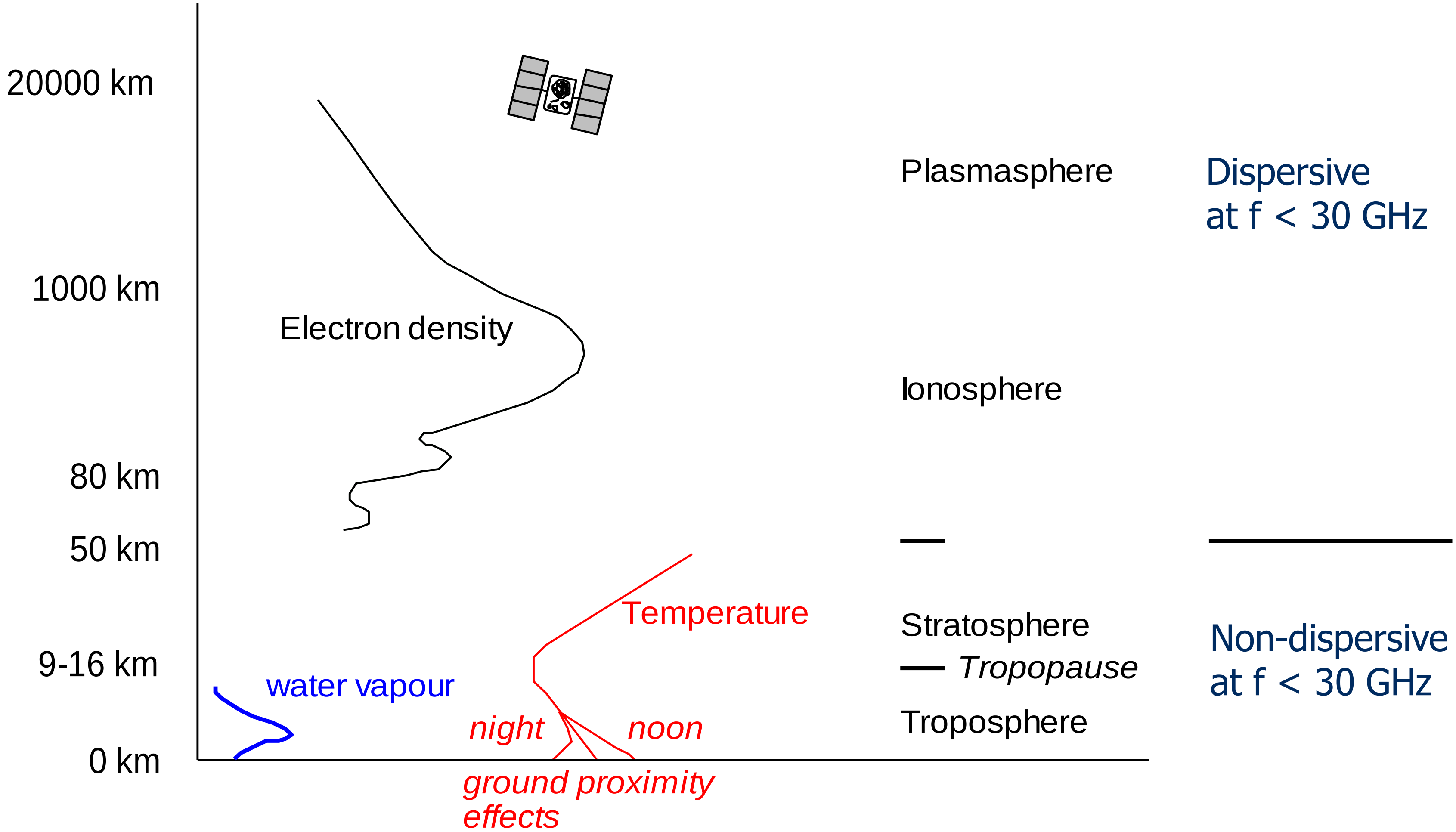
IGS orbits, clocks and Ionosphere map



L1 pseudo range measurements

But ionosphere free l.c. of carrier phase can give cm accuracy…

Atmosphere of the Earth



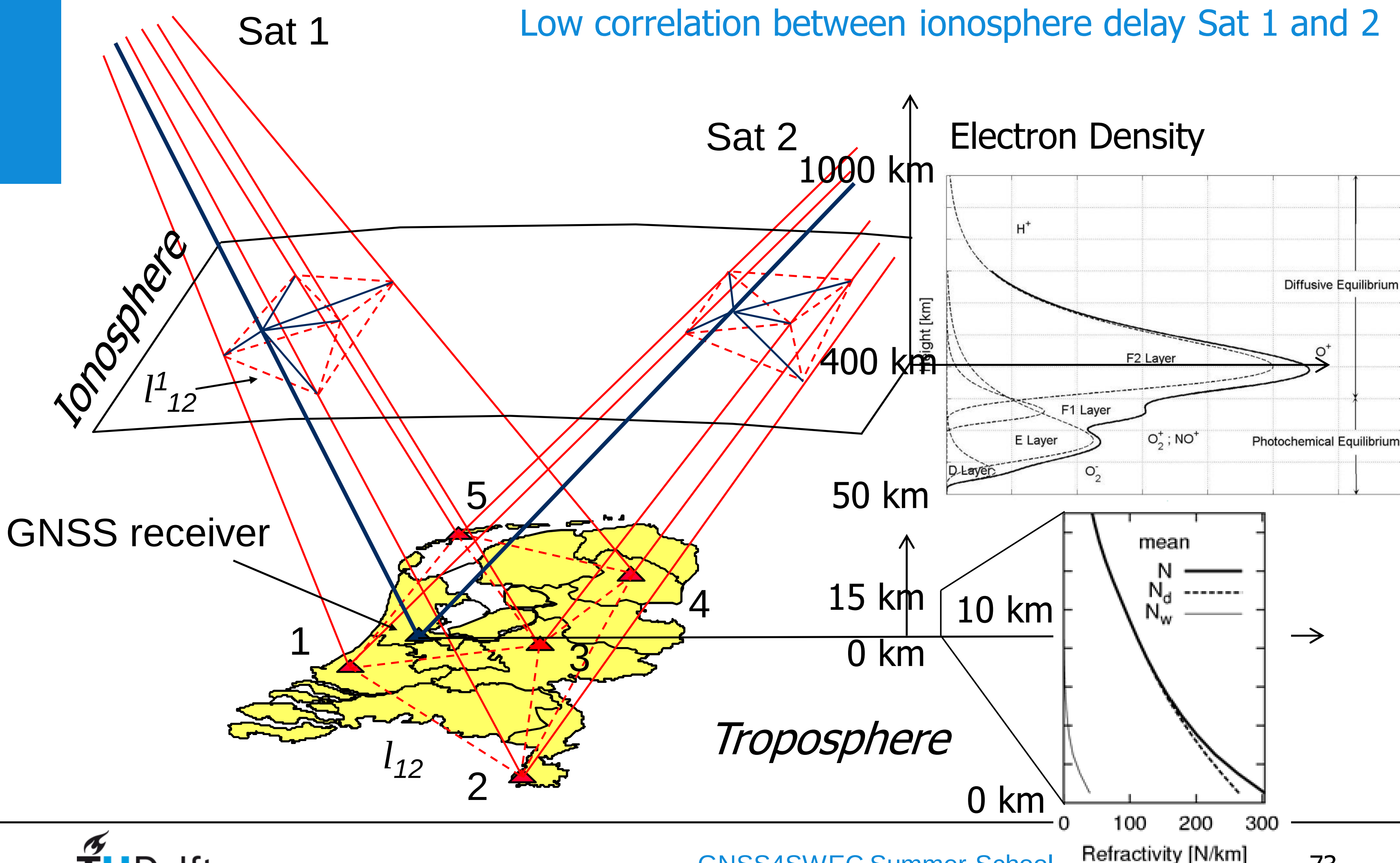
Atmospheric Delay Errors

	ionosphere	troposphere
height	50 – 1000 km	0 – 16 km
variability	diurnal, seasonal, solar cycle (11 yr), solar flares	relatively low
zenith delay	meters – tens of meters	2.3 – 2.6 m (sea level)
obliquity factor $el=30^o$	1.8	2
obliquity factor $el=15^o$	2.5	4
obliquity factor $el=3^o$	3	10
modeling error (zenith) ***)**	0.2 - >10 m	5 – 10 cm (no met. data)
dispersive	**yes**	no

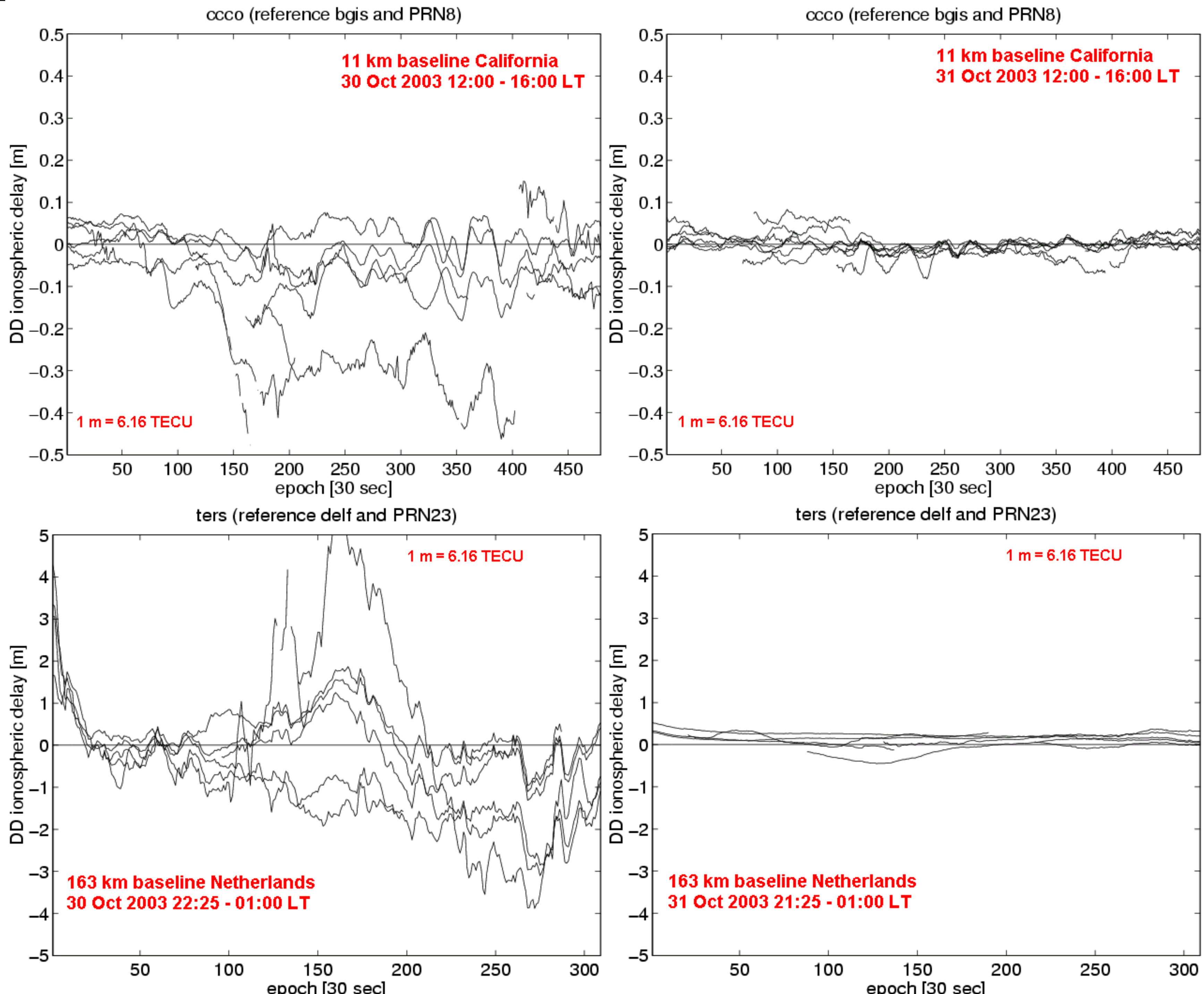
all values are approximate, depending on location and circumstances

***) mm-cm errors in case of ionosphere free linear combination, zenith delay estimation, or relative positioning**

Atmospheric Delay Errors



Double Difference Ionosphere delay



Ionosphere

- For GNSSS (L-band) signals **ionosphere is dispersive** :
  - refractive index depends on frequency of signal
  - different phase (carrier) and group (code) velocities, v_p and v_g , with refractive index

$$n_p = \frac{c}{v_p}$$

$$n_g = \frac{c}{v_g} = n_p + f\,\frac{dn_p}{df}$$

modulated carrier wave → superposition of a **group** of waves of different frequencies

- ➢ Results in frequency dependent delays
- ➢ Results in phase delay being negative (advance!)

Ionosphere: Total Electron Content

- Propagation speed depends on **total electron content** (**TEC**) = number of electrons in tube of 1 m^2 from receiver to satellite

$$\mathrm{TEC} = \int_S^R n_e(l)\,dl \quad [\mathrm{TECU}]$$

1 TECU (TEC Unit) = 10^{16} electrons / m^2

with $n_e(l)$ the electron density along the path

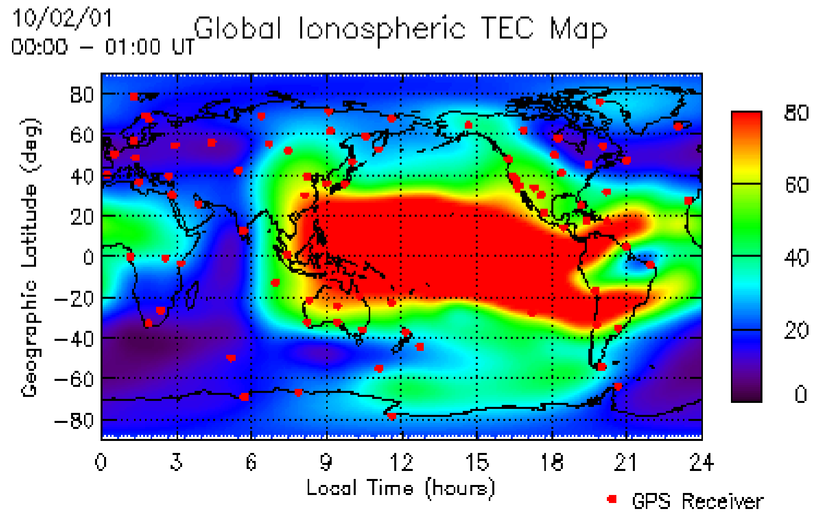
- Can be computed from dual frequency GNSS observations

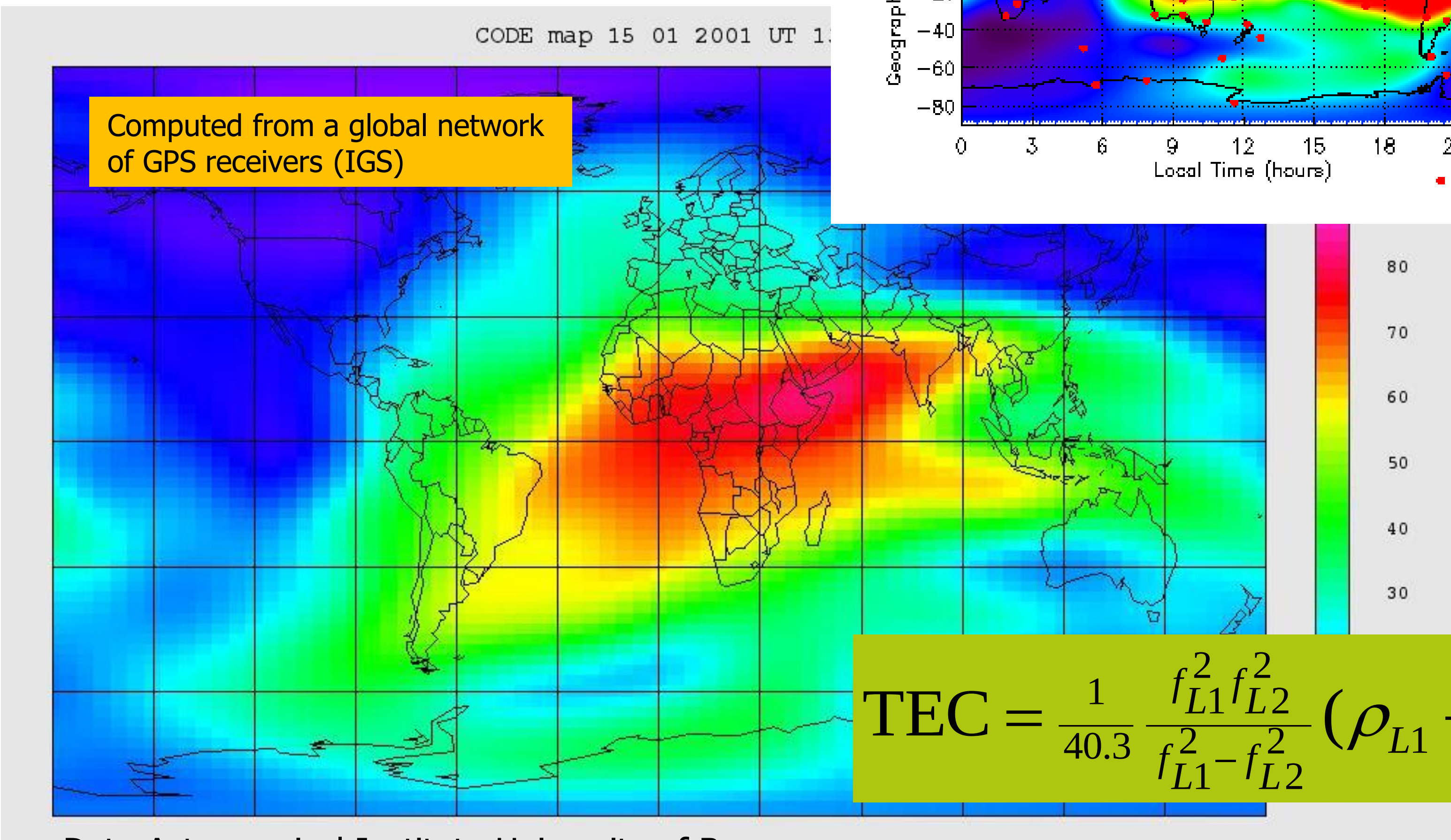
$$\mathrm{TEC} = \frac{1}{40.3}\frac{f_{L1}^2 f_{L2}^2}{f_{L1}^2 - f_{L2}^2}(\rho_{L1} - \rho_{L2})$$

$$I_{L1} = \frac{40.3 \cdot \mathrm{TEC}}{f_{L1}^2}$$

$$I_{L2} = \frac{40.3 \cdot \mathrm{TEC}}{f_{L2}^2} = \frac{f_{L1}^2}{f_{L2}^2} I_{L1}$$

Ionospheric TEC map (from GPS)





Computed from a global network of GPS receivers (IGS)

$$\mathrm{TEC} = \frac{1}{40.3}\,\frac{f_{L1}^2 f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\left(\rho_{L1} - \rho_{L2}\right)$$

Data Astronomical Institute University of Berne

Ionosphere: how to deal with it?

- apply broadcast model (reduction 50 – 70%)
- use TEC maps from IGS (.25 – 1 m error, ok for code)
- ionosphere-free combination (dual-frequency receiver required!)

Our choice

$$\rho_{IF} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2}\rho_{L1} - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\rho_{L2}$$

$$I_{L1} = \frac{40.3 \cdot \text{TEC}}{f_{L1}^2}$$

$$I_{L2} = \frac{40.3 \cdot \text{TEC}}{f_{L2}^2} = \frac{f_{L1}^2}{f_{L2}^2} I_{L1}$$

$$\rho_{IF} = r + c\left[\delta t_u - \delta t^s\right] + \cancel{I} + T + \varepsilon_{\rho_{IF}}$$

bias removed;
noise increased

- relative positioning (short baselines only)

Ionosphere-free linear combination

- Ionosphere-free linear combination (GPS L1 & L2 freq.)

$$\rho_{IF} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2}\rho_{L1} - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\rho_{L2} \approx 2.546\rho_{L1} - 1.546\rho_{L2}$$

- Ionospheric delay removed to first order (residual effects ~ mm)
- St.dev. ionosphere free linear combination factor 3 larger (argh..)

$$\sigma_{\rho_{IF}} \approx \sqrt{2.546^2\sigma_{\rho_{L1}}^2 + 1.546^2\sigma_{\rho_{L2}}^2} \approx 3\sigma_{\rho}$$

- Works for pseudo range (code) and carrier-phase
- For carrier-phase the ionosphere free linear combination has a wavelength of 0.6 cm (or 10.6 cm if we know the wide-lane)

Ionosphere-free linear combination

- St.dev. of the ionosphere free linear combination factor 3 larger
- Errors on L1 are multiplied by a factor 2.5, and errors on L2 by a factor -1.5 . This implies
  - Frequency dependent errors could easily be amplified
  - Especially errors with opposite sign on L1 and L2 are amplified
  - Extra care needs to be taken for antenna and other instrumental delays in case different antenna and receiver types are combined
- Using L5 instead of L2 results in a small improvement
- Using three frequencies instead of two also helps *)

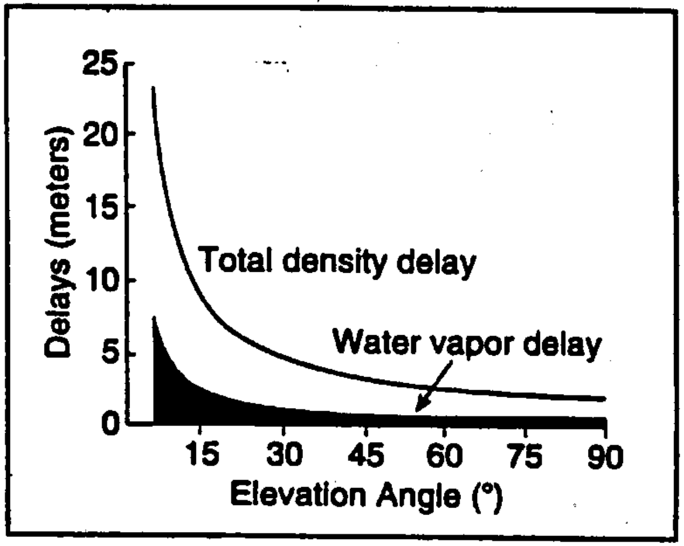
*) Three frequencies could also be used to eliminate higher order effects, but this is not recommended because this will increase the sensitivity to frequency dependent instrumental errors only further. If higher order effects are a problem use a correction algorithm.

Troposphere delay error

- Troposphere: 9 km (@poles) – 16 km (@equator)
- Propagation speed lower than in free space -> apparent range is longer (2.5m@90^0 – 25m@5^o)
- Non-dispersive, i.e. refraction does not depend on frequency, same phase and group velocities, i.e.

$$T_{\rho_{L1}} = T_{\rho_{L2}} = T_{\phi_{L1}} = T_{\phi_{L2}} = T$$

- Dry gases and water vapor



How to deal with it:

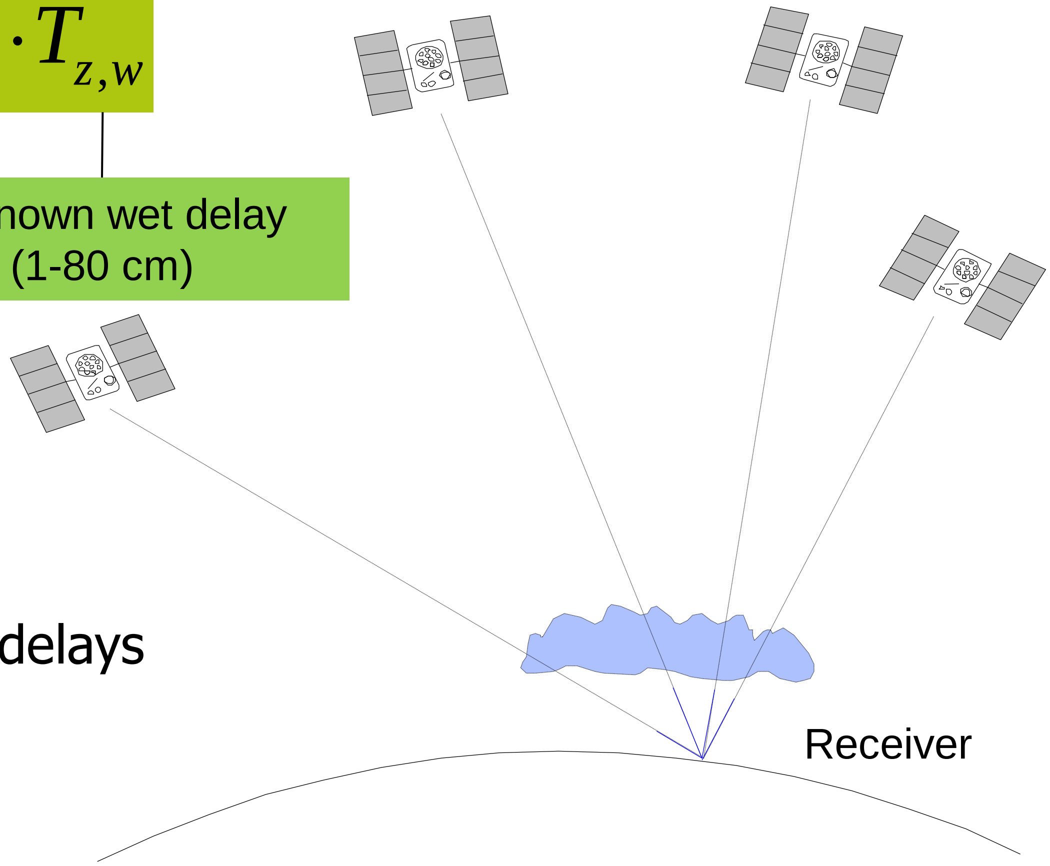
1. Tropospheric delay models (zenith delay error 5 – 10 cm)
2. Extra unknown parameter — Our choice
3. Relative Positioning (short baselines)

Troposphere delay estimation

$$T = m_d(el) \cdot T_{z,d} + m_w(el) \cdot T_{z,w}$$

Computed hydrostatic delay (few mm accuracy)

Unknown wet delay (1-80 cm)



m_d, m_w mapping functions

$T_{z,d}, T_{z,w}$ zenith troposphere delays

el elevation

Extra unknown parameter (position + clock + zenith delay)

Recipe for millimeter GNSS

Recipe

- Use carrier phase data → estimate extra ambiguities
- Use ionosphere free linear combination
- Estimate extra zenith delay parameter → our "little" bonus !!

- Use IGS orbits and clocks , or **Precise Point Positioning**
- Use IGS orbits and form baselines **Network processing**

- May have to include some other corrections, such as phase windup, tides, loading corrections, …

This changes the observation equations drastically …

how to determine the GNSS position with mm accuracy,
and get atmospheric delay as a bonus

Fundamentals of GNSS Processing

GNSS CARRIER PHASE SOLUTION

Code and Carrier Phase Equation

Ionosphere is dispersive; coefficient is 0 for ionosphere free l.c.

b

Constant in time!

$$\rho_{Li} = \left\| \mathbf{x}^{(k)} - \mathbf{x} \right\| + \frac{f_1^2}{f_i^2} I_{L1} + T + c\delta t_r - c\delta t^s + \varepsilon_{\rho_{Li}}$$

$$\Phi_{Li} = \left\| \mathbf{x}^{(k)} - \mathbf{x} \right\| - \frac{f_1^2}{f_i^2} I_{L1} + T + c\delta t_r - c\delta t^s + \lambda_{Li} A_{Li} + \varepsilon_{\Phi_{Li}}$$

$$\sigma_\rho \approx 20-100 \text{ cm} , \quad \sigma_\phi \approx 1-2 \text{ mm}$$

Note: All parameters depend on t, except for ambiguities!

Note: Ambiguities must be resolved to take advantage of high precision phase measurements

Linerized code observation equations

$$(-\mathbf{1}^{(k)})^T = \left[\frac{x^{(k)}{}_0 - x_0}{\left\|\mathbf{x}^{(k)}{}_0 - \mathbf{x}_0\right\|} \quad -\frac{y^{(k)}{}_0 - y_0}{\left\|\mathbf{x}^{(k)}{}_0 - \mathbf{x}_0\right\|} \quad -\frac{z^{(k)}{}_0 - z_0}{\left\|\mathbf{x}^{(k)}{}_0 - \mathbf{x}_0\right\|} \right]$$

Unit direction vector (receiver to satellite)

Single satellite:

$$\delta\rho^{(k)} = (-\mathbf{1}^{(k)})^T \delta\mathbf{x} + \delta b + \tilde{\varepsilon}_\rho^{(k)}$$

$$\delta\rho^{(k)} = \rho^{(k)} - \rho_0^{(k)}$$

Observed minus computed

All satellites in view:

$$\boldsymbol{\delta\rho} = \begin{bmatrix} \delta\rho^{(1)} \\ \delta\rho^{(2)} \\ \vdots \\ \delta\rho^{(m)} \end{bmatrix} = \begin{bmatrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \delta b \end{bmatrix} + \tilde{\boldsymbol{\varepsilon}}_\rho$$

$m \times 1$ $\quad$ $\boldsymbol{G}$ ($m \times 4$) $\quad$ 4×1

$$\delta\mathbf{x} = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}$$

$$\delta b = b - b_0$$

Linerized carrier phase equation (1)

Single satellite:

$$\delta\Phi_{Li}^{(k)} = (-\mathbf{1}^{(k)})^T \delta\mathbf{x} + \delta b + \lambda_{Li} A_{Li}^{(k)} + \tilde{\varepsilon}_{\Phi}^{(k)}$$

$$\Phi_{Li}^{(k)} = \Phi_{Li}^{(k)} - \Phi_{Li\,0}^{(k)}$$

Observed minus computed

All satellites in view (single epoch):

$$\boldsymbol{\delta\Phi} = \begin{bmatrix} \delta\Phi^{(1)} \\ \delta\Phi^{(2)} \\ \vdots \\ \delta\Phi^{(m)} \end{bmatrix} = \Bigg[\underbrace{\begin{matrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{matrix}}_{\boldsymbol{G}} \;\; \underbrace{\begin{matrix} \lambda_{Li} & & & \\ & \lambda_{Li} & & \\ & & \ddots & \\ & & & \lambda_{Li} \end{matrix}}_{\lambda_{Li}\mathbf{I}}\Bigg] \begin{bmatrix} \delta\mathbf{x} \\ \delta b \\ A_{Li}^{(1)} \\ A_{Li}^{(2)} \\ \vdots \\ A_{Li}^{(m)} \end{bmatrix} + \tilde{\boldsymbol{\varepsilon}}_{\Phi}$$

$m \times 1$ $\quad$ $m \times 4$ $\quad$ $m \times m$ $\quad$ $(4+m) \times 1$

Linerized carrier phase equation (2)

The equations on the previous slide are rank-defect. Proof:

$$\underbrace{\begin{bmatrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{bmatrix}}_{\boldsymbol{G}} \underbrace{\begin{bmatrix} \lambda_{Li} & & & \\ & \lambda_{Li} & & \\ & & \ddots & \\ & & & \lambda_{Li} \end{bmatrix}}_{\lambda_{Li}\mathbf{I}} \begin{bmatrix} \mathbf{0} \\ -\lambda_{Li} \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$m \times 4$ $\quad m \times m$ $\quad (4+m) \times 1$ $\quad m \times 1$

I.e. columns are linear dependent, therefore, no unique solution.
We have to re-parameterize the unknown parameters.

Linerized carrier phase equation (3)

Single satellite:

$$\delta\Phi_{Li}^{(k)} = (-\mathbf{1}^{(k)})^T \delta\mathbf{x} + \delta b + \lambda_{Li} A_{Li}^{(k)} + \tilde{\varepsilon}_{\Phi}^{(k)}$$

All satellites in view (single epoch, rank defect solved):

$$\boldsymbol{\delta\Phi} = \begin{bmatrix} \delta\Phi^{(1)} \\ \delta\Phi^{(2)} \\ \vdots \\ \delta\Phi^{(m)} \end{bmatrix} = \underbrace{\begin{bmatrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{bmatrix}}_{\boldsymbol{G}} \underbrace{\begin{bmatrix} \\ \lambda_{Li} & & \\ & \ddots & \\ & & \lambda_{Li} \end{bmatrix}}_{\lambda_{Li}\bar{\mathbf{I}}} \begin{bmatrix} \delta\mathbf{x} \\ \delta b + \lambda_{Li} A_{Li}^{(1)} \\ A_{Li}^{(2)} - A_{Li}^{(1)} \\ \vdots \\ A_{Li}^{(m)} - A_{Li}^{(1)} \end{bmatrix} + \tilde{\boldsymbol{\varepsilon}}_{\Phi}$$

$m \times 1$ $\quad m \times 4$ $\quad m \times (m-1)$ $\quad (3+m) \times 1$

Linerized carrier phase equation (4)

Re-parameterized unknowns:

Biased receiver clock error

$$\delta\bar{b} = \delta b + \lambda_{Li} A_{Li}^{(1)}$$

Single difference ambiguities:

$$A_{Li}^{(1k)} = A_{Li}^{(k)} - A_{Li}^{(1)}$$

Linerized carrier phase equation (5)

Single satellite:

$$\delta\Phi_{Li}^{(k)} = (-\mathbf{1}^{(k)})^T \delta\mathbf{x} + \delta b + \lambda_{Li} A_{Li}^{(k)} + \tilde{\varepsilon}_{\Phi}^{(k)}$$

All satellites in view (single epoch, rank defect solved):

$$\boldsymbol{\delta\Phi} = \underbrace{\begin{bmatrix} \delta\Phi^{(1)} \\ \delta\Phi^{(2)} \\ \vdots \\ \delta\Phi^{(m)} \end{bmatrix}}_{m \times 1} = \begin{bmatrix} \underbrace{\begin{matrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{matrix}}_{\substack{\boldsymbol{G} \\ m \times 4}} & \underbrace{\begin{matrix} \\ \lambda_{Li} & & \\ & \ddots & \\ & & \lambda_{Li} \end{matrix}}_{\substack{\lambda_{Li}\overline{\mathbf{I}} \\ m \times (m-1)}} \end{bmatrix} \underbrace{\begin{bmatrix} \delta\mathbf{x} \\ \delta\overline{b} \\ A_{Li}^{(12)} \\ \vdots \\ A_{Li}^{(1m)} \end{bmatrix}}_{(3+m) \times 1} + \tilde{\boldsymbol{\varepsilon}}_{\Phi}$$

Linerized carrier phase equation (6)

Single satellite with Troposphere Zenith Delay (ZTD) correction:

$$\delta\Phi_{Li}^{(k)} = (-\mathbf{1}^{(k)})^T \delta\mathbf{x} + \delta b + m_w^{(k)}\delta T + \lambda_{Li} A_{Li}^{(k)} + \tilde{\varepsilon}_{\Phi}^{(k)}$$

All satellites in view (single epoch, rank defect solved, ZTD):

$$\boldsymbol{\delta\Phi} = \underset{m\times 1}{\begin{bmatrix} \delta\Phi^{(1)} \\ \delta\Phi^{(2)} \\ \vdots \\ \delta\Phi^{(m)} \end{bmatrix}} = \begin{bmatrix} \underbrace{\begin{matrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(m)})^T & 1 \end{matrix}}_{\substack{\boldsymbol{G} \\ m\times 4}} & \underbrace{\begin{matrix} m_w^{(1)} \\ m_w^{(2)} \\ \vdots \\ m_w^{(m)} \end{matrix}}_{\substack{\mathbf{m} \\ m\times 1}} & \underbrace{\begin{matrix} & & \\ \lambda_{Li} & & \\ & \ddots & \\ & & \lambda_{Li} \end{matrix}}_{\substack{\lambda_{Li}\bar{\mathbf{I}} \\ m\times(m-1)}} \end{bmatrix} \underset{(4+m)\times 1}{\begin{bmatrix} \delta\mathbf{x} \\ \delta\bar{b} \\ \delta T \\ A_{Li}^{(12)} \\ \vdots \\ A_{Li}^{(1m)} \end{bmatrix}} + \tilde{\boldsymbol{\varepsilon}}_{\Phi}$$

Linerized carrier phase equation (7)

Dependent on time

Independent of time

static / kinematic

$$
\underbrace{\begin{bmatrix} \delta\Phi^{(1)} \\ \delta\Phi^{(2)} \\ \vdots \\ \delta\Phi^{(m)} \end{bmatrix}}_{\boldsymbol{\delta\Phi}_i}
=
\begin{bmatrix}
\underbrace{\begin{matrix} (-\mathbf{1}^{(1)})^T \\ (-\mathbf{1}^{(2)})^T \\ \vdots \\ (-\mathbf{1}^{(m)})^T \end{matrix}}_{\bar{\mathbf{G}}_i}
&
\underbrace{\begin{matrix} 1 & m_w^{(1)} \\ 1 & m_w^{(2)} \\ \vdots & \vdots \\ 1 & m_w^{(m)} \end{matrix}}_{\mathbf{B}_i}
&
\underbrace{\begin{matrix} \lambda_{Li} & & \\ & \ddots & \\ & & \lambda_{Li} \end{matrix}}_{\lambda_{Li}\bar{\mathbf{I}}}
\end{bmatrix}
\begin{bmatrix}
\delta\mathbf{x} \\
\left.\begin{matrix} \delta\bar{b} \\ \delta T \end{matrix}\right\} \delta\bar{\mathbf{b}}_i \\
\left.\begin{matrix} A_{Li}^{(12)} \\ \vdots \\ A_{Li}^{(1m)} \end{matrix}\right\} \bar{\mathbf{a}}_{Li}
\end{bmatrix}
+ \tilde{\boldsymbol{\varepsilon}}_\Phi
$$

$m \times 1$ $\quad m \times 3$ $\quad m \times 2$ $\quad m \times (m-1)$ $\quad (4+m) \times 1$

Linerized carrier phase equation (8)

Multiple epochs, static position:

$$
\begin{bmatrix} \delta\mathbf{\Phi}_1 \\ \delta\mathbf{\Phi}_2 \\ \vdots \\ \delta\mathbf{\Phi}_N \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{G}}_1 & \lambda_{Li}\bar{\mathbf{I}} & \mathbf{B}_1 & & & \\ \bar{\mathbf{G}}_2 & \lambda_{Li}\bar{\mathbf{I}} & & \mathbf{B}_2 & & \\ \vdots & \vdots & & & \ddots & \\ \bar{\mathbf{G}}_N & \lambda_{Li}\bar{\mathbf{I}} & & & & \mathbf{B}_N \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \bar{\mathbf{a}}_{Li} \\ \delta\bar{\mathbf{b}}_1 \\ \delta\bar{\mathbf{b}}_2 \\ \vdots \\ \delta\bar{\mathbf{b}}_N \end{bmatrix} + \tilde{\boldsymbol{\varepsilon}}_{\mathbf{\Phi}}
$$

POS

AMB

CLK & ZTD ↓

$Nm \times 1$ $\quad Nm \times 3$ $\quad Nm \times (m-1)$ $\quad Nm \times 2N$ $\quad (3+m\text{-}1+2N) \times 1$

Multiple epochs, kinematic position, is left as exercise

Zenith Delay parameter(s)

- In the previous slide corrections to the a-priori Zenith Total delay (ZTD) appear once per epoch, however,
- The actual ZTD is quite smooth compared to the GNSS sample rate (seconds to minutes), therefore,
- The zenith delay correction is either solved

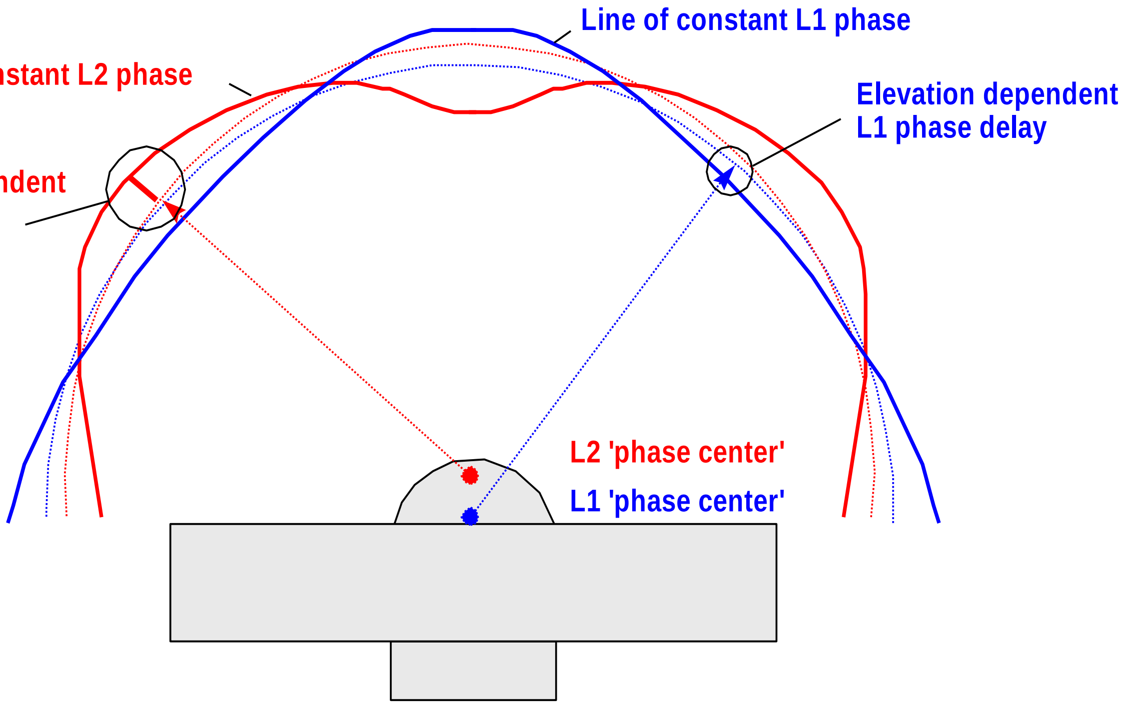
1. in batches (e.g. one parameter per hour), or,
2. as piecewise polynomials, or,
3. as random-walk or Markov process.(specifying a power spectral density)

- In addition to ZTD gradient parameters can be estimated as well. Gradient parameters are typically estimated only once or twice per day.

Remarks and observations

- Receiver clock error is based
- Only single difference ambiguities can be estimated
- Clock parameter is kinematic parameter (white noise)
- Troposphere delay is a dynamic parameter (random walk, Markov process) or piecewise polynomial
- Position parameter is static parameter
  - Site displacement effects (tides, loading,…) must be corrected
  - For well behaving permanent stations, with well known position and velocity, the position parameter may be constrained or even removed from the equations
- Height, receiver clock, and troposphere delay are highly correlated, especially for high elevation cutoff's
- Tropospheric Zenith Delay estimation is very sensitive to changes in the antenna phase patterns. Therefore, antenna phase "center" variations must be modeled carefully.

Antenna Phase Center Variations (1)



The phase center is a theoretical concept; it does not actually exist; but, a ‘phase center’ can be computed by fitting a sphere to lines of constant phase.
Calibrations for the phase center variations are given in the form of a table in elevation and azimuth.

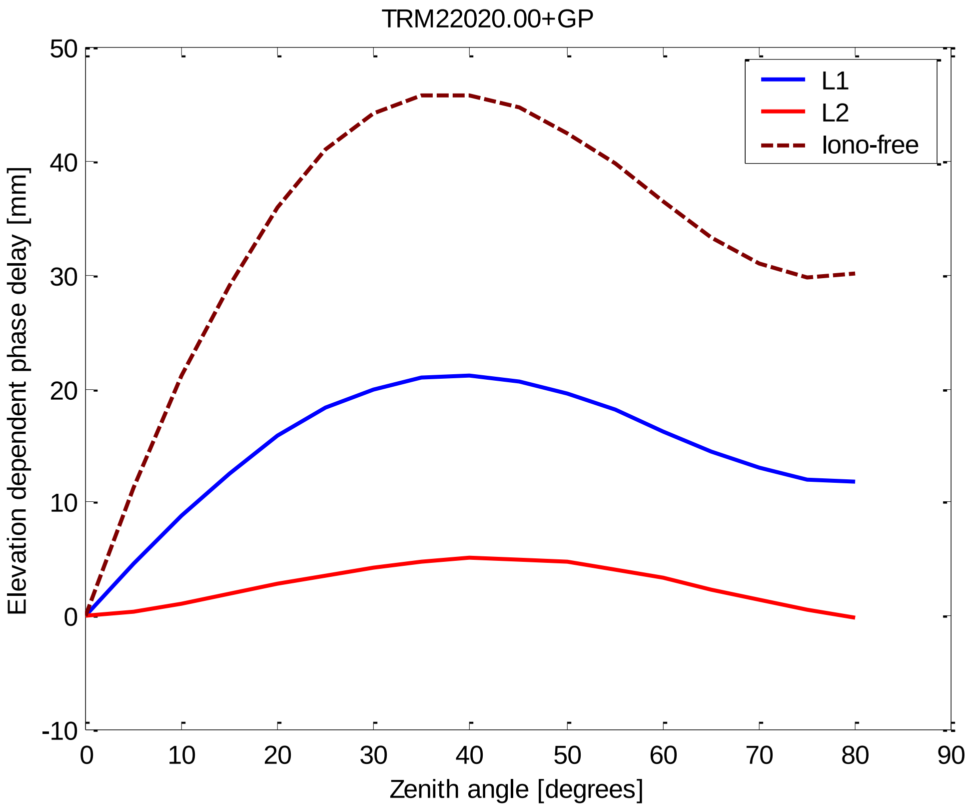
Antenna Phase Center Variations (2)

Elevation dependent phase delays for a real antenna.

L1 and L2 elevation dependent delay are not the same.

Effect in the order of mm-cm.

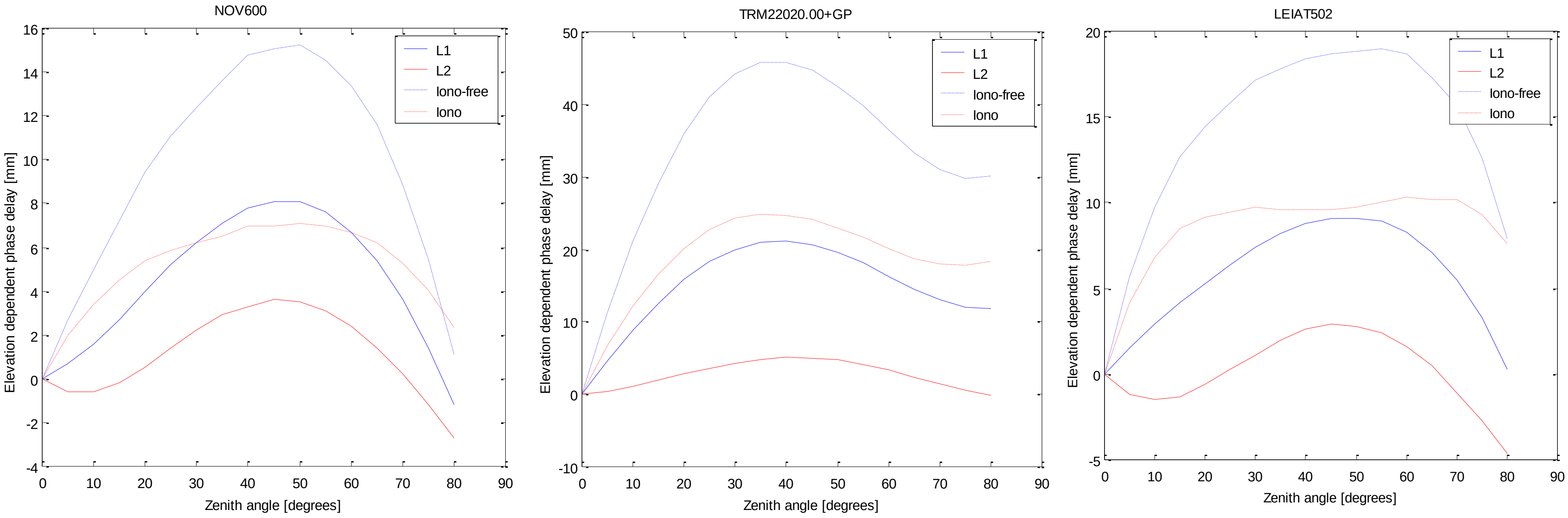
<u>The effect is amplified in the ionosphere free linear combination (max 4.6 cm for this antenna)</u>



Antenna phase center variations (3)

Patterns for different antennae

- Type calibrations
- Individual calibrations

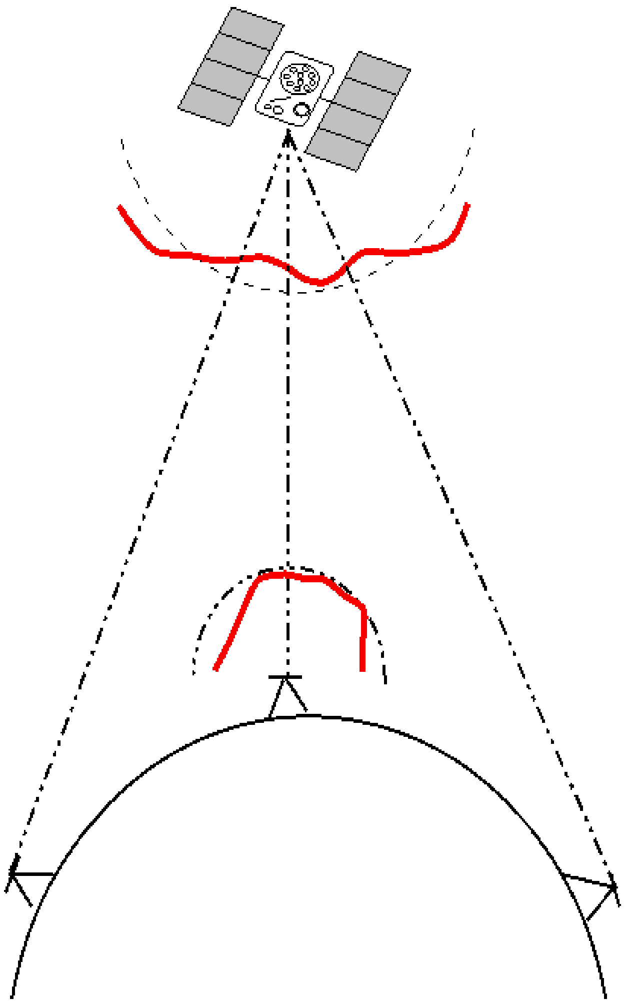


Antenna Phase Center Variations (4)

Absolute calibrations:
- Satellite antenna (block/type dependent)
- Receiver antenna (type/individual)

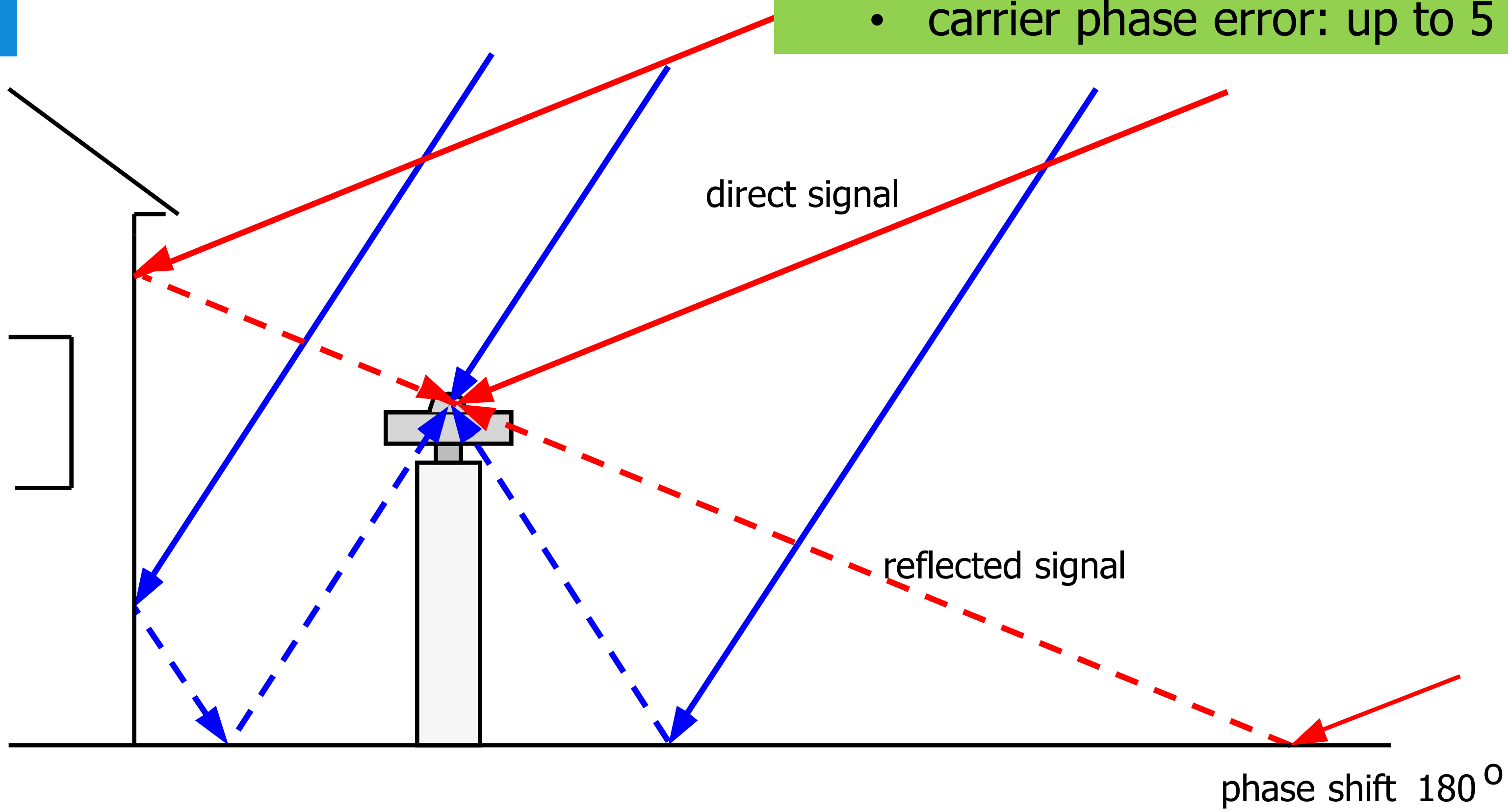
Relative calibrations:
- With respect to 'standard' AoA (JPL) Dorne-Margolin chokering antenna
- Chokering antenna was assumed `perfect' (but is not, but by coincidence this absorbed the effect of not using satellite antenna calibrations
- now depricated (or only for short baselines)

Satellite antenna delays have been calibrated using GPS measurements from the IGS network; scale factor fixed to SLR value



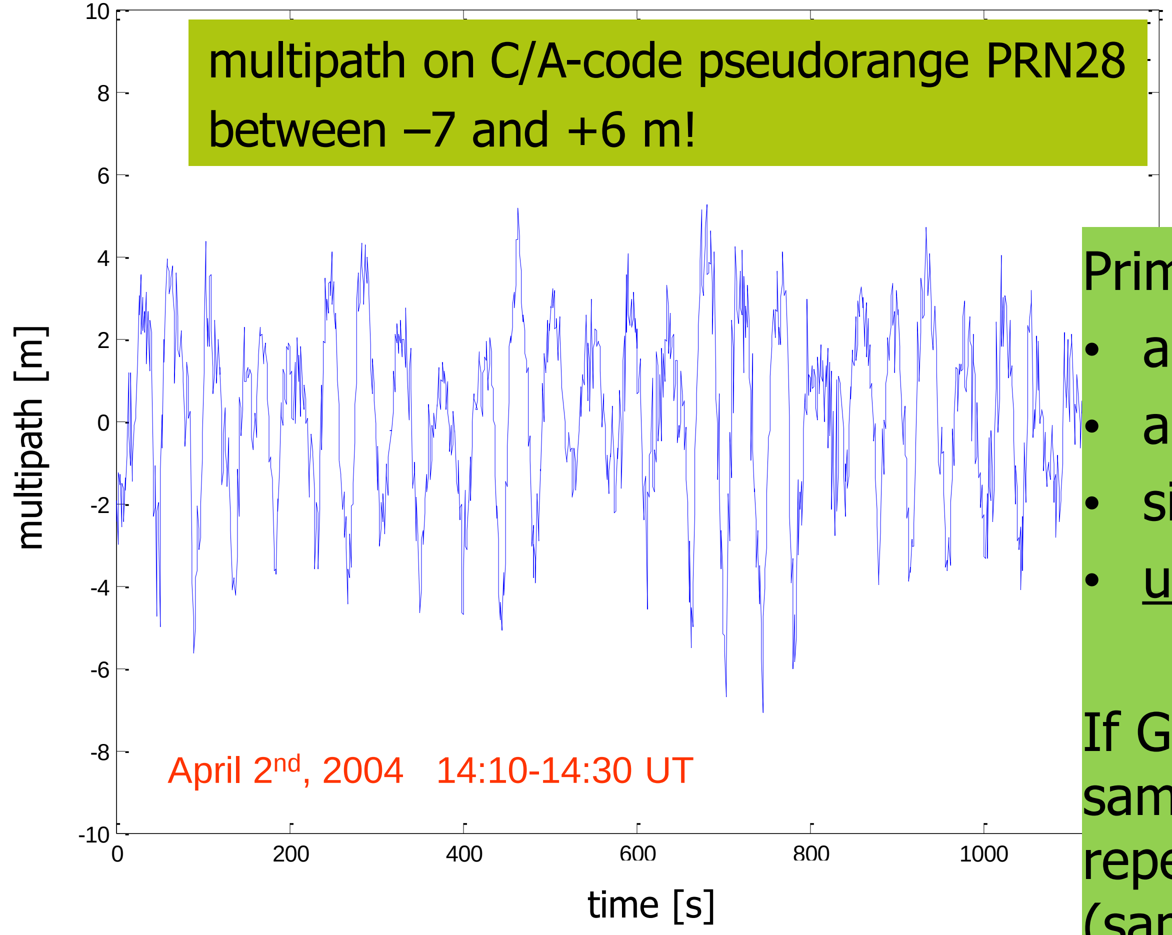
Multipath

- Signal arrives via two or more paths
- Reflected signals have different path length and interfere with direct signal
- Systematic errors (do not average out):
  - pseudorange: up to tens of meters
  - carrier phase error: up to 5 cm



Multipath: example

$Mc = C1 - 4.092*L1 + 3.092*L2$ [m]



multipath on C/A-code pseudorange PRN28 between –7 and +6 m!

Primary defense:

- antenna location
- antenna design
- signal processing
- use carrier phase

If GPS receiver is static: same multipath pattern repeats after 23h56m (same orbit)

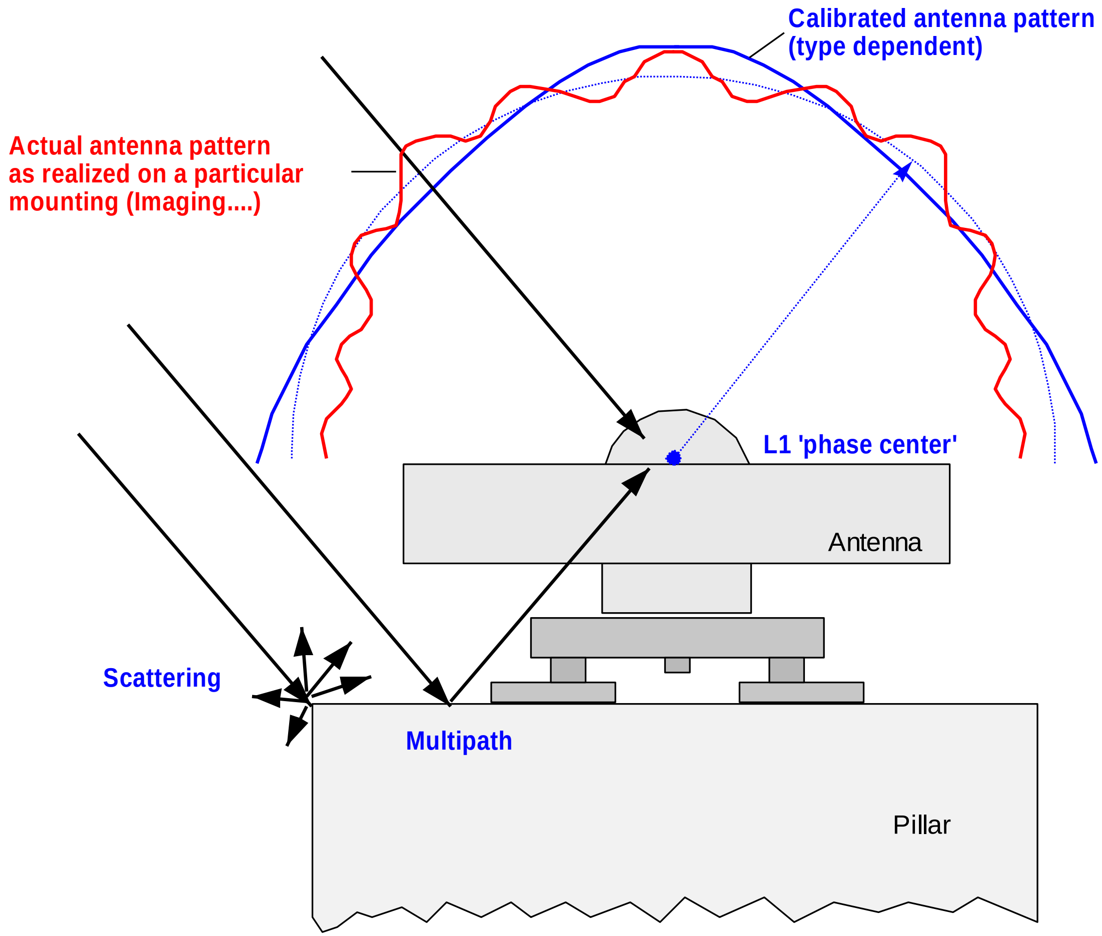
Near field effects

The elevation and azimuth dependent phase delays are affected by nearby objects (in the near field), such as the monument and antenna radome.

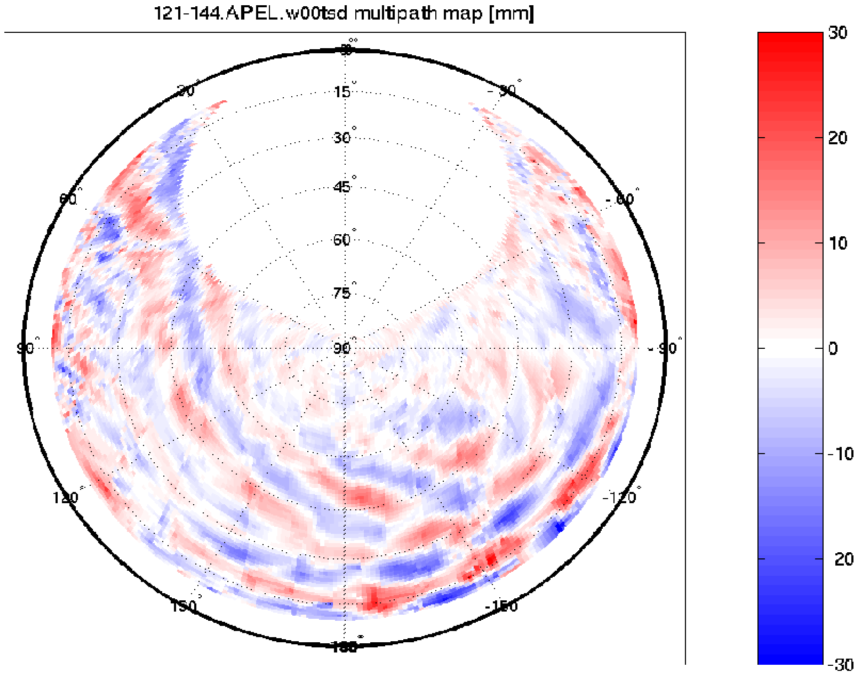
The result is that the actual antenna pattern is different from the type dependent calibration.

These effects can be studied by stacking residuals (observed minus computed) after estimating the station position and zenith delays. This will reveal some patterns, but not all, so it cannot be a replacement for proper calibration procedures.

This is especially important for Slant Delay Estimation.



Residual stacking – Carrier Phase Multipath Maps



Example of a multipath map for Apeldoorn, May 1-24 2003 (1x1 degree bins in an equal area projection)

To correct GPS observations for multipath (affects all parameters, including slant delays) [iteration]

To quantify and visualize multipath/antenna effects:

- Polar plots (w/ interpolation -> see example)
- Elevation dependent phase delay plots (next)
- Elevation dependent standard deviation plots (next)

To re-weight observations

What remains after correction with multipath maps are the slant tropospheric delays!

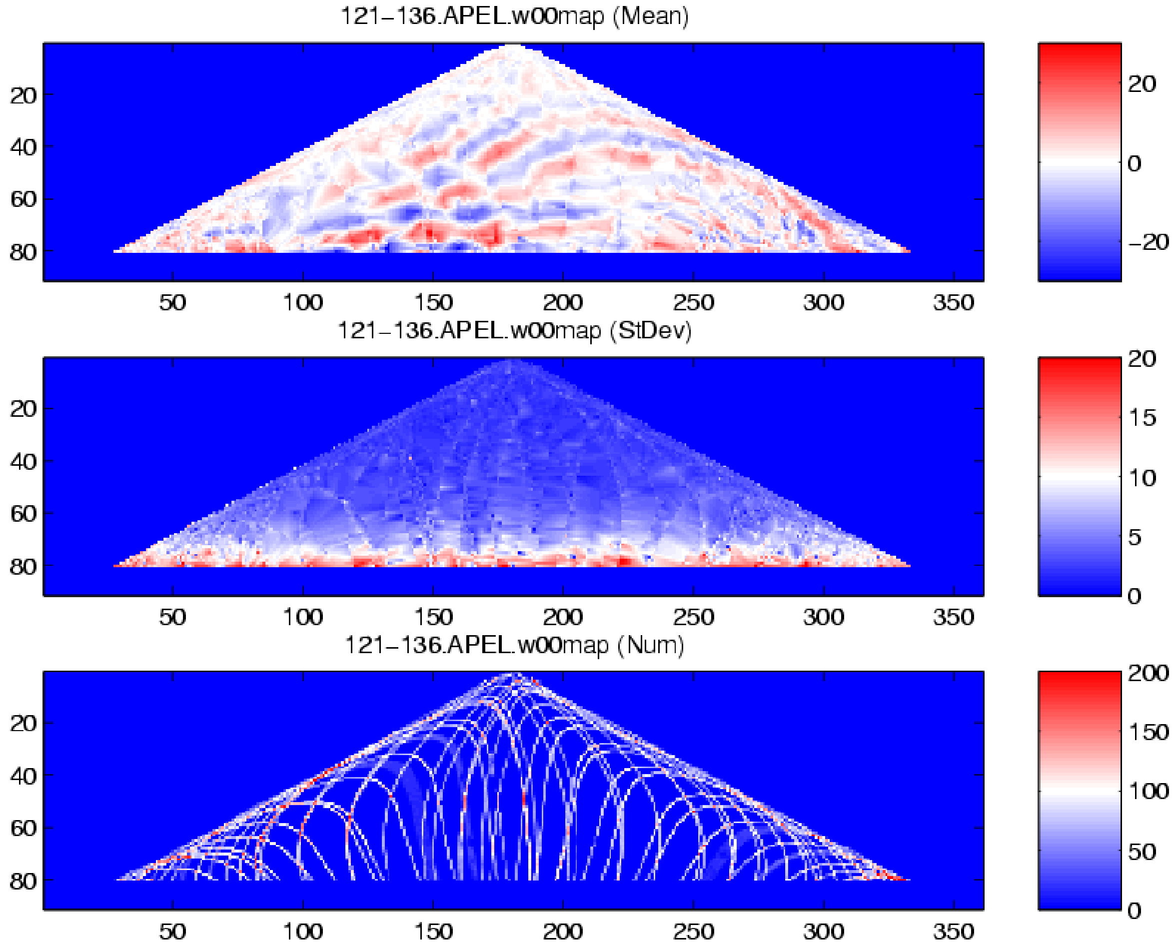
Residual stacking - Implementation

mean

RMS

(sum of squares)

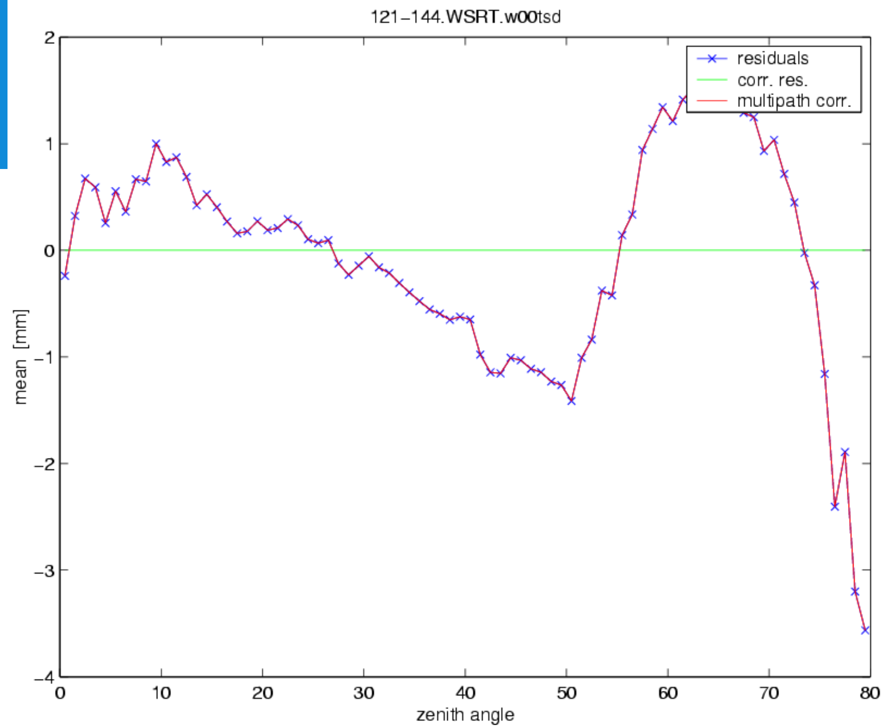
count



Equal area projection

Interpolated for visualization purposes

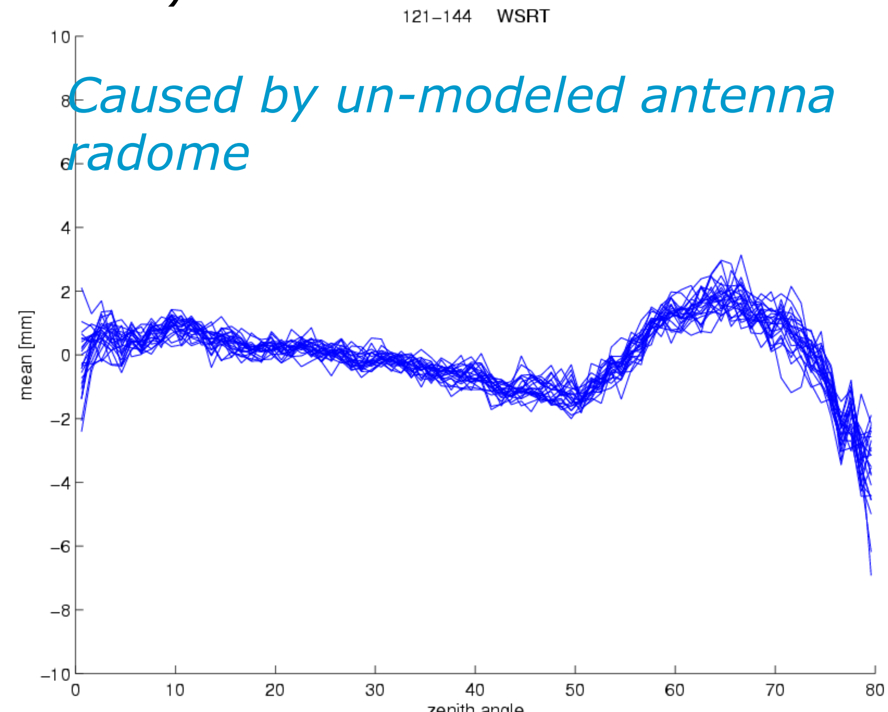
Residual stacking – Elevation dependent delay



Multiday average

Example of a elevation dependent delay map for Westerbork (WSRT), May 1-24 2003 (1 degree bins)

Caused by un-modeled antenna radome

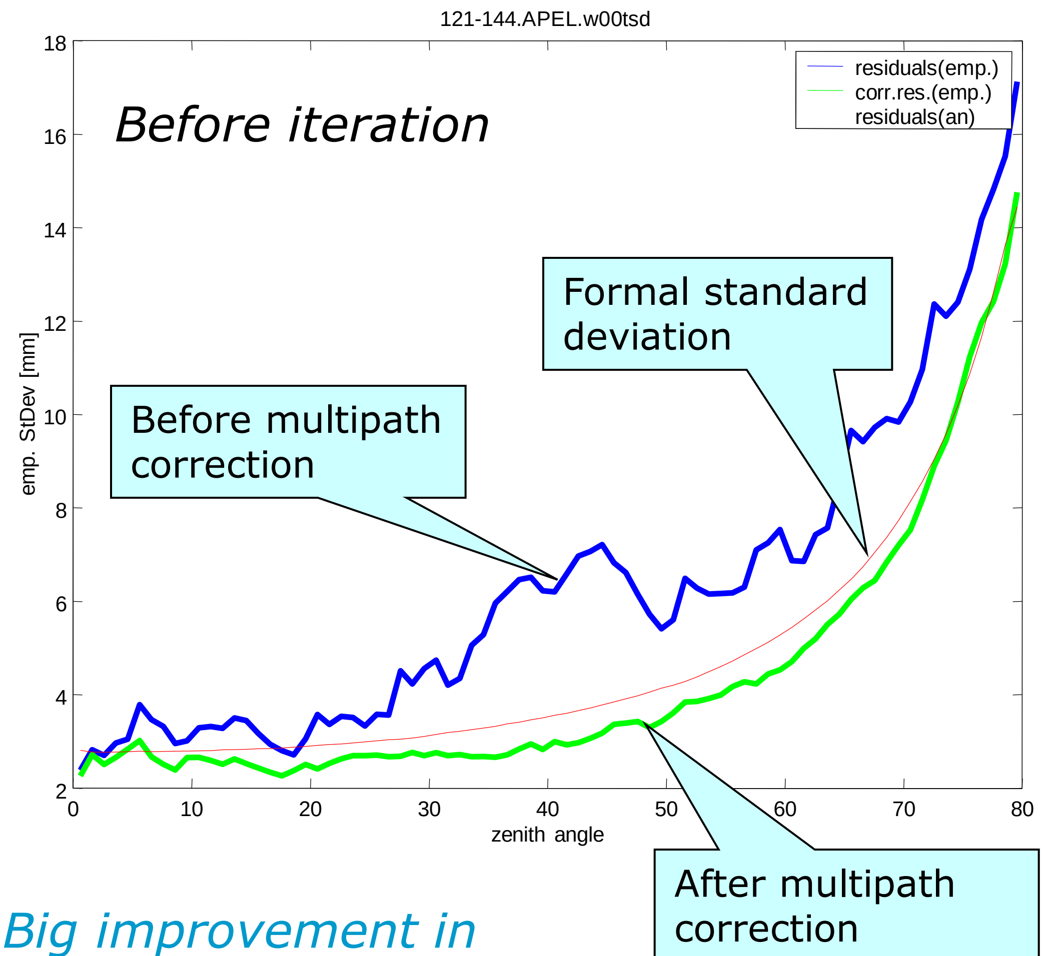


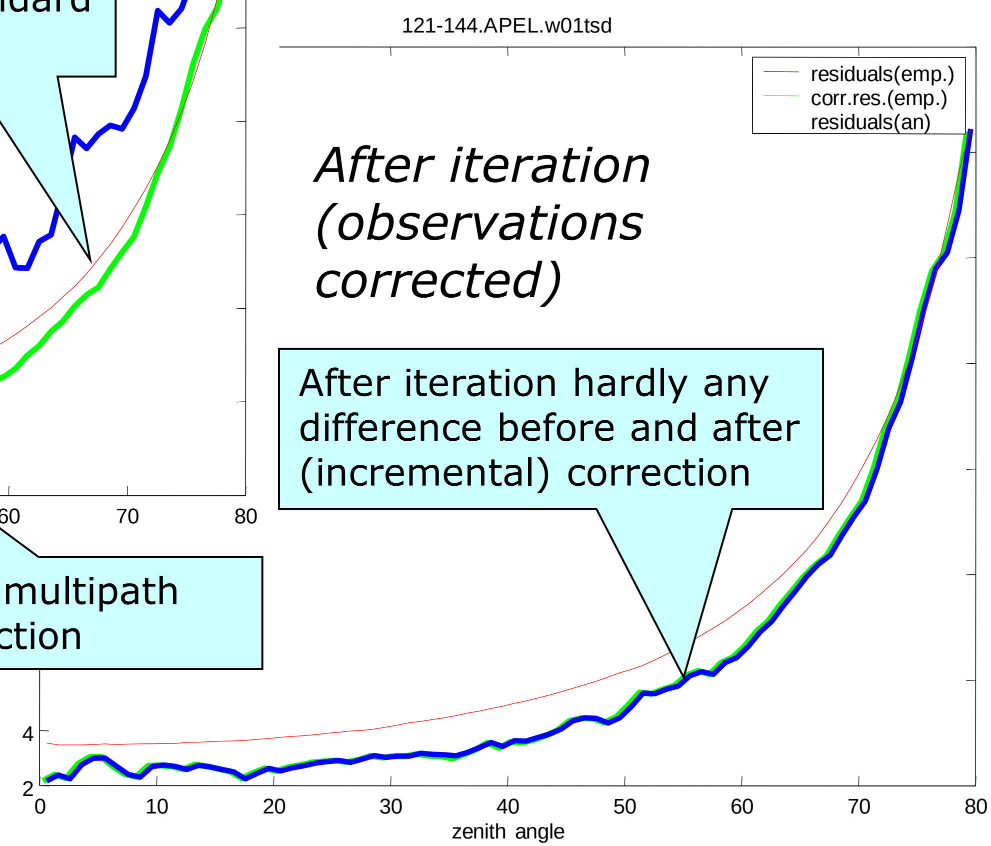
Daily results

Can reveal some patterns, but not a*cos(z) and b*cos(z)$^{-1}$

Residual stacking – Elevation dependent weighting

Example of elevation dependent standard deviation for Apeldoorn, May 1-24 2003

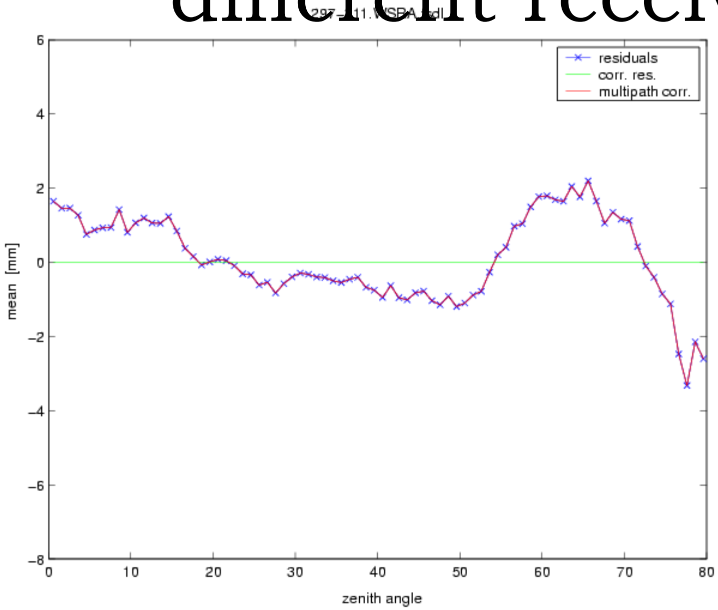


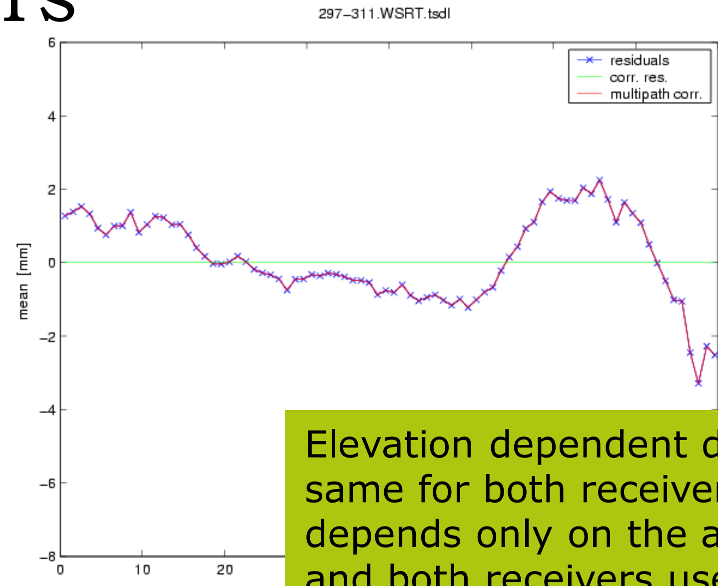


Big improvement in standard deviation

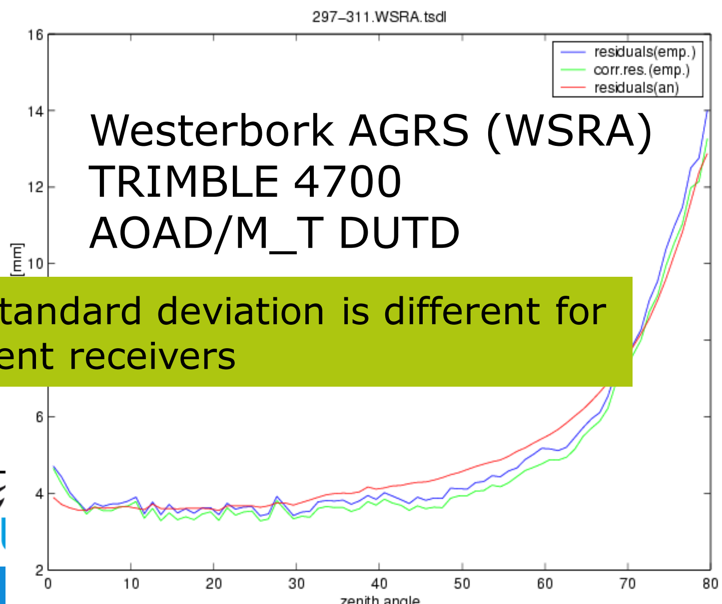
TU Delft

Residual Stacking - Same antenna; different receivers



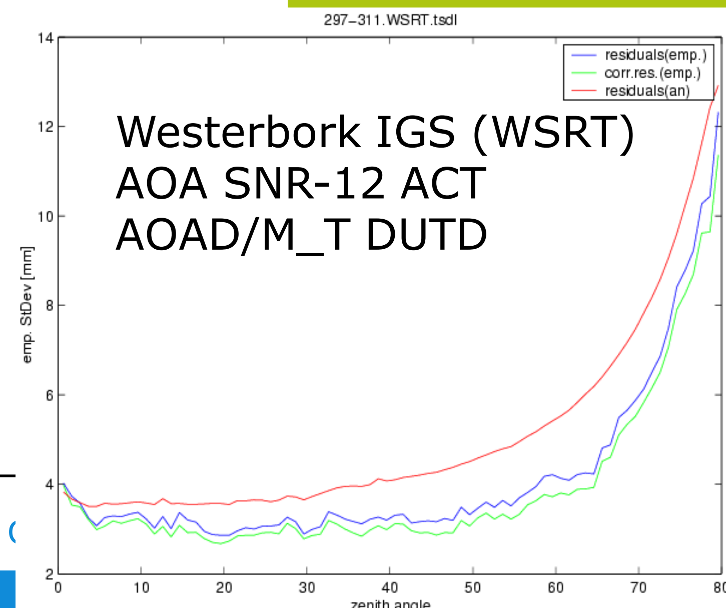


Elevation dependent delay same for both receivers: depends only on the antenna, and both receivers use the same antenna



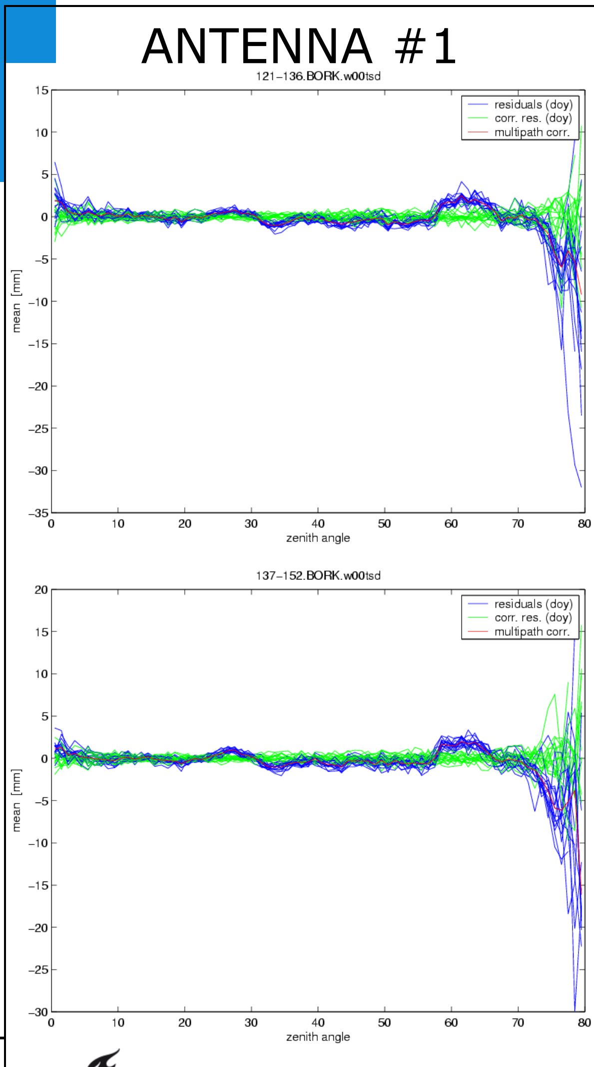
Westerbork AGRS (WSRA)
TRIMBLE 4700
AOAD/M_T DUTD

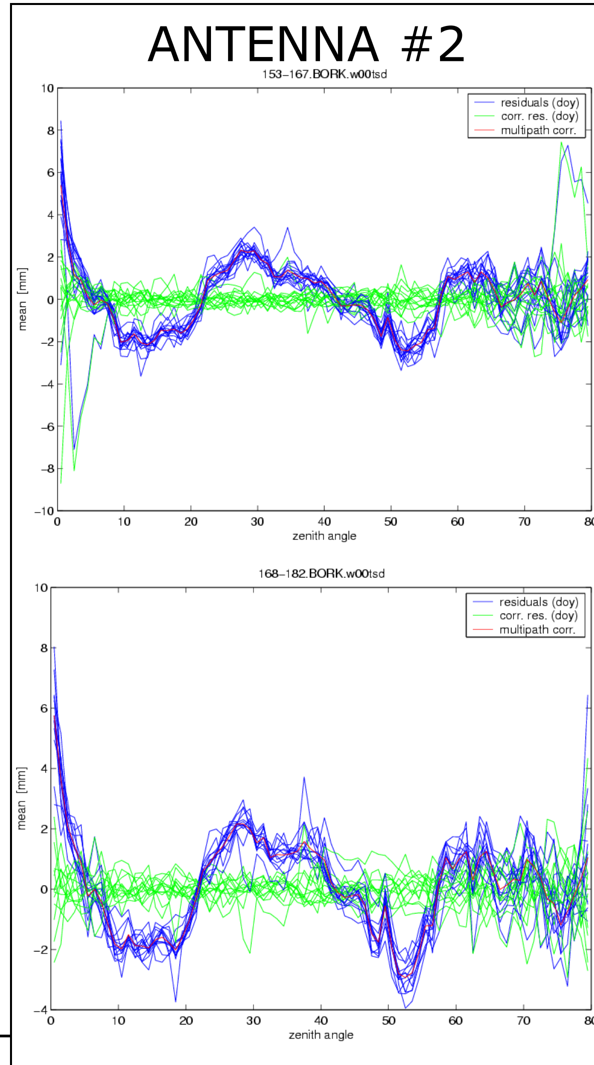
The standard deviation is different for different receivers

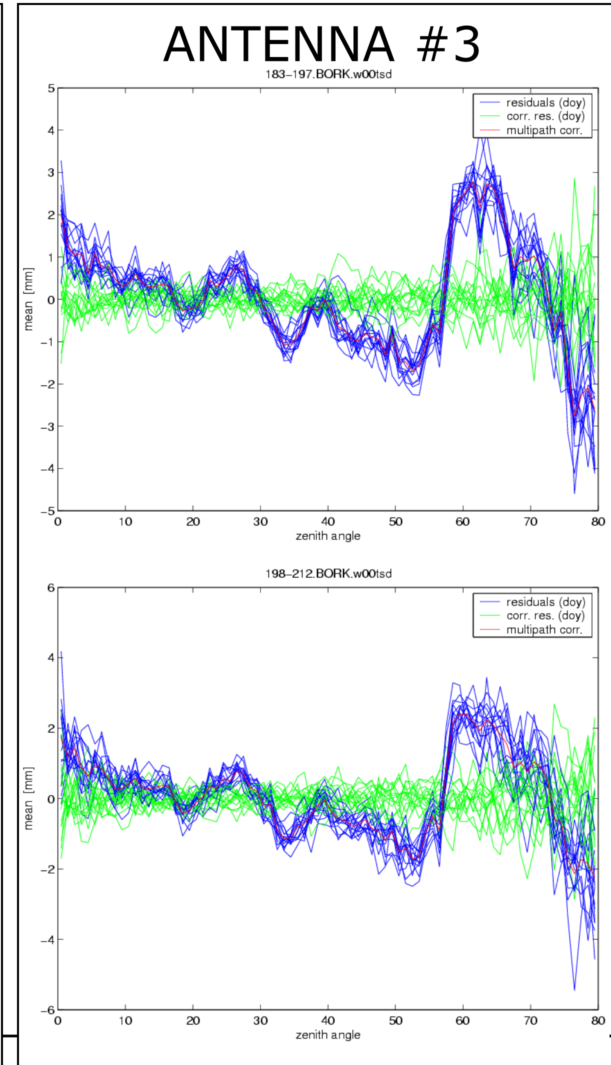


Westerbork IGS (WSRT)
AOA SNR-12 ACT
AOAD/M_T DUTD

Residual stacking: impact of antenna changes (BORK)
3 month period - subdivided in six 2-week periods







“Beware of the dog” notice

If something changes with elevation, beware and pay attention, it will affect your ZTD and height estimates.

Some examples so far…

- Antenna elevation (and azimuth) dependent phase delay
  - Satellites
  - Receiver antenna calibrations
  - Near field effects (monument, radome, …)
- Mapping function for the troposphere parameter
- Elevation dependent weighting
- …

Slant Delays

Slant delays can be computed in two ways:

1. From estimated ZTD and gradiient parameters
2. Slant delay = m.f. * ZTD + l.s.-residual + corr. Multipath

Mapping function (m.f.) is needed also in the 2nd slant delay approach to separate receiver clock error from atmospheric delay:

- receiver clock error is the same for every l.o.s. observation
- delay increases with the zenith angle

Slant delays of the same receiver at each epoch are

- Biased with the same “residual” receiver clock error
- Correlated

This should be well understood for assimilation

What are slant delays?

"Slant delays" are a way of preprocesing the GPS data:

- Satellite orbits, station position, clocks and antenna effects are removed
- As an extra service multipath effects are removed
- Easy access to raw data for meteorologists

Several functions of observations have already been used

- receiver and satellite clocks: "mean delay"
- cos(e) and 1/cos(e) functions (height, ZTD)
- ....

Optimal interval for Slant and Zenith Delays?

  - Zenith delay every 20 minutes?
  - Slant delay every 5 minutes (or even every epoch)?

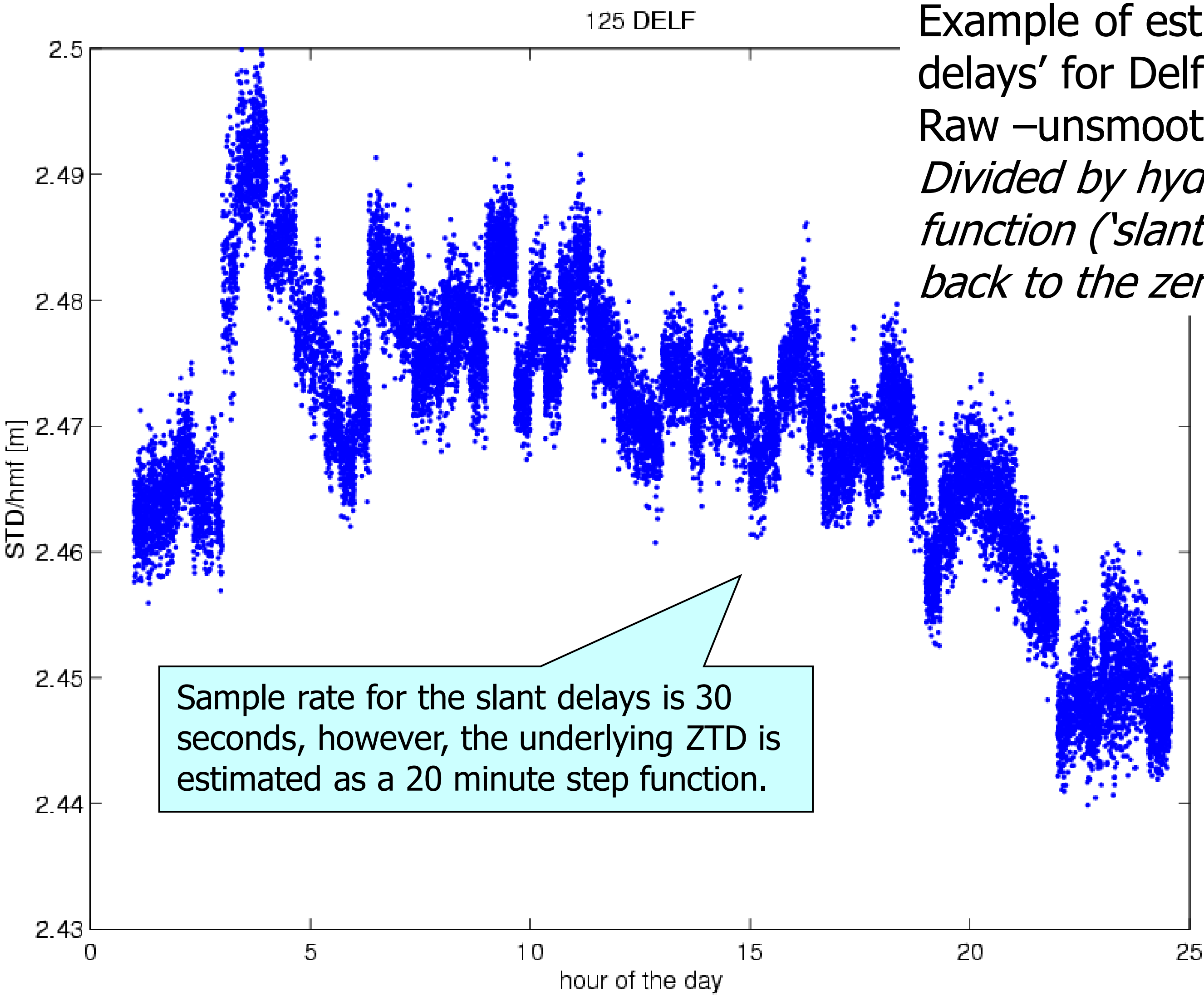
Rationale for Slant Delays

Compared to ZTD, with slants delays …

- it is known which part of the atmosphere is sampled (the sky is not sampled homogeneously)
- information on non-isotropic delays and gradients in the atmosphere become available
- there are simply more observations and they are much 'closer' to the raw GPS data
- could include more low elevation observations
- data can be (re-)processed more intelligently down the line:

  - Might be useful to investigate assimilation (or tomography) of single or double differenced slant delays, or include additional parameters
  - ZTD can be recomputed from slant delays, using different mapping functions, or even re-doing receiver clock estimation

Example of ‘slant delays’



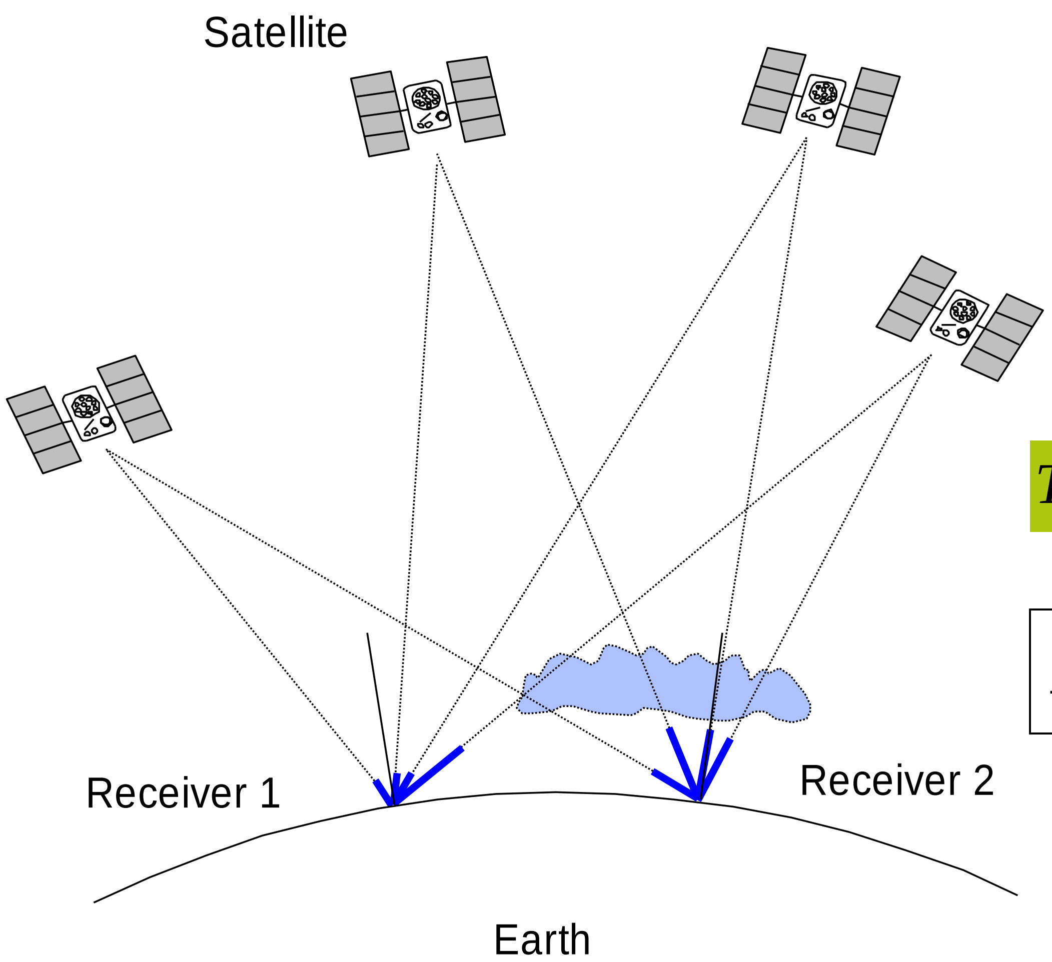
Example of estimated ‘slant delays’ for Delft on May 5 2003; Raw –unsmoothed- product; *Divided by hydrostatic mapping function (‘slant delay’ mapped back to the zenith direction)*

From delay to column water vapour

Fundamentals of GNSS Processing

INTEGRATED WATER VAPOUR (IWV) PROCESSING

Integrated Water Vapour from GNSS



$$T = M_d(e) * T_d^z + M_w(e) * T_w^z$$

Observed delay

computed hydrostatic delay

Known mapping function

Unknown tropospheric zenith wet delay

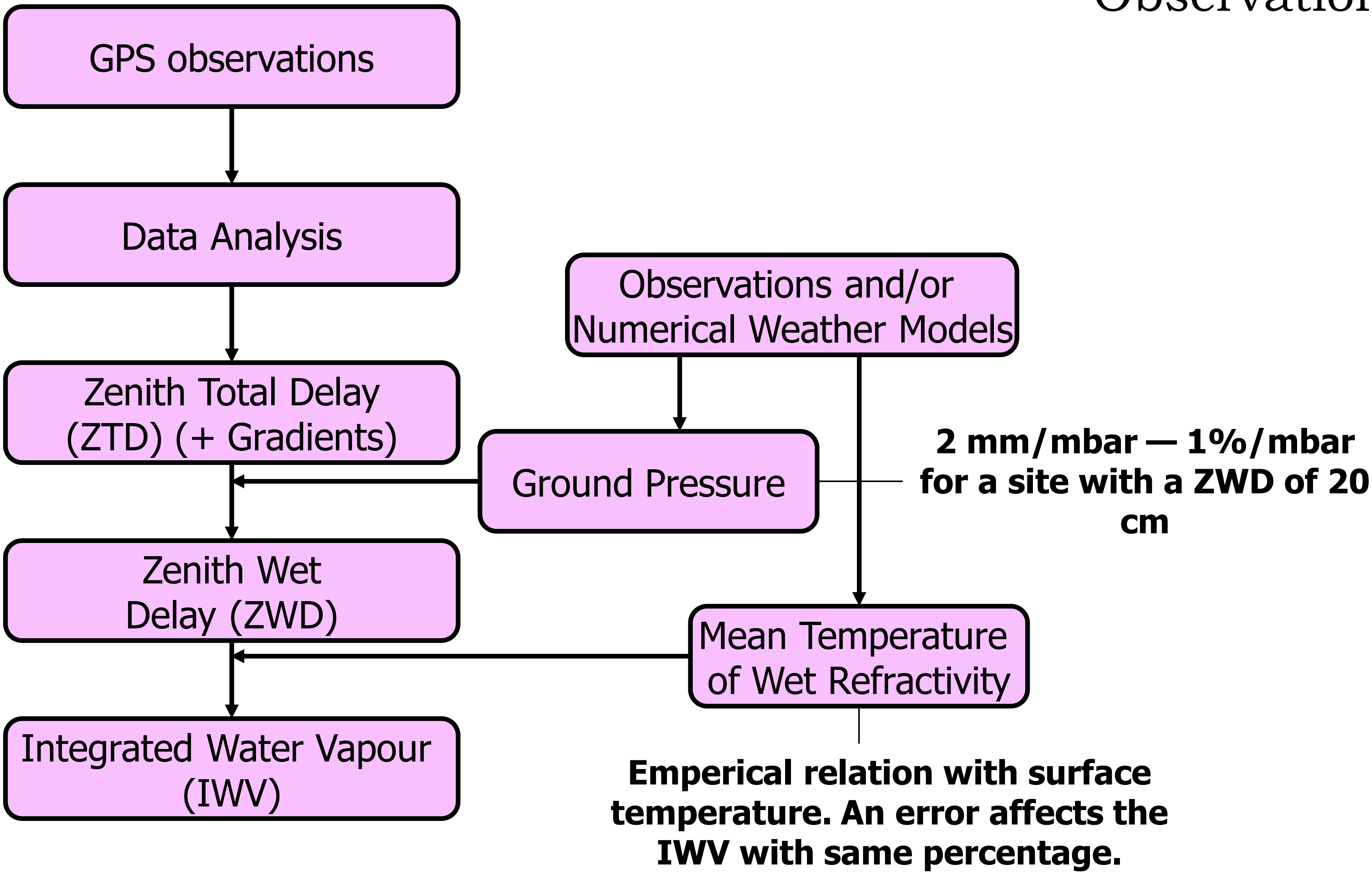
Integrated Water Vapour Estimation

Integrated water vapour estimation comprises the following steps:

1. Ionosphere free linear combinations from dual frequency GPS
2. Compute a-priori values for Zenith Hydrostatic Delay from surface pressure values (observed or from model), multiply with hydrostatic mapping function and subtract from ionosphere free l.c.
3. Introduce the Zenith Wet Delay (ZWD) as an extra parameter in the GNSS processing, and estimate it along with position, receiver clock error and other parameters. The wet mapping function is used as coefficient in the design matrix to relate the observed slant delay to the vertical wet delay
4. Convert ZWD into Integrated Water Vapour using $Q(T_m)$ or $Q(T_s)$.

Some applications (e.g. NWP) use ZTD=ZHD+ZWD instead.

GPS Integrated Water Vapour Observation



Conversion to Integrated Water Vapour

The zenith delay (T_{ZTD}), estimated by GPS, is converted to integrated water vapour (IWV) using surface pressure and temperature readings

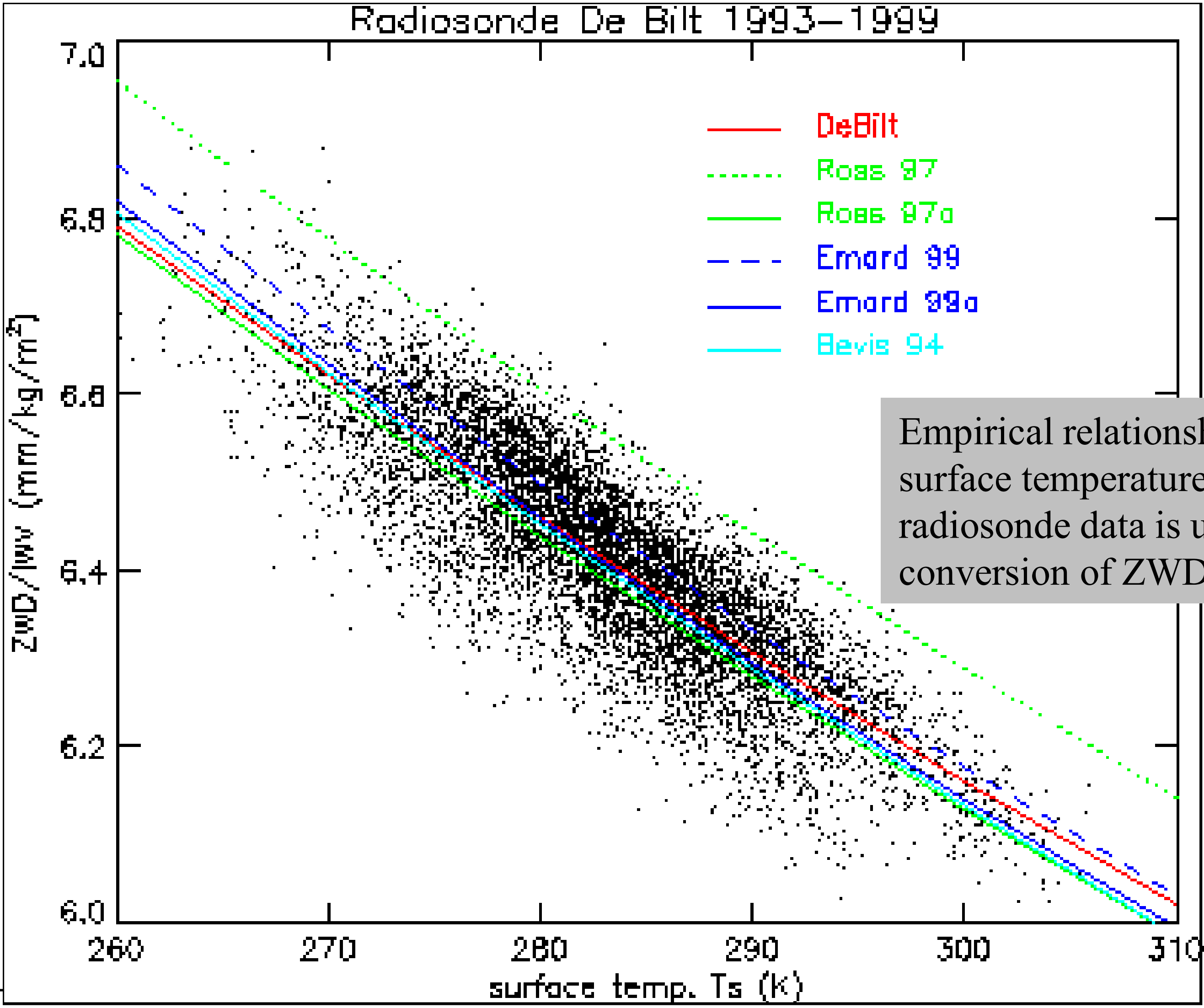
$$IWV = \frac{1}{Q(T_m)}(\hat{T}_{ZTD} - T_{ZHD}(P_s, \varphi, h)) \quad , \quad T_m \approx 70.2 + 0.72 T_s$$

- T_{ZHD} the hydrostatic zenith delay, calculated from surface pressure P_s, station latitude and height
- conversion factor $Q(T_m)$ is ~6.5 (function of mean temp)
- unit for integrated water vapour (IWV) is kg/m^2
- accuracy is 1-2 kg/m^2

Problem: surface temperature $T_s \rightarrow T_m$, i.e. $Q(T_s) \rightarrow Q(T_m)$

For numerical weather prediction we may use estimated ZTD instead!

$Q(T_s)$ from radiosonde De Bilt 1993-1999



Empirical relationship $Q(T_s)$ with surface temperature; line fitted to radiosonde data is used in the conversion of ZWD to IWV

To network or to PPP …

Fundamentals of GNSS Processing

PRECISE POINT POSITIONING VS NETWORKS

Time is up…

- In the next lecture Jan Douse will compare PPP and Network approaches

- In the appendices you will find some extra information on
  - Differential (relative) GNSS using carrier phase measurements
  - The International GNNS Service
  - Background information on troposphere delay and mapping functions

122

For more information see Navipedia
http://www.navipedia.net/index.php/Main_Page

Fundamentals of GNSS Processing

QUESTIONS?

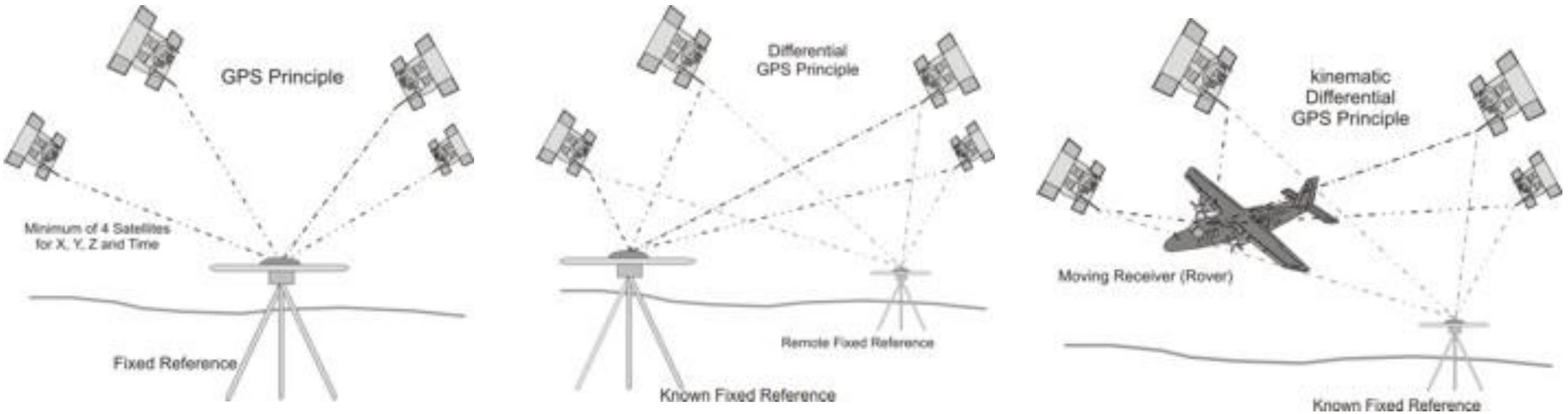
Many GNSS errors are eliminated or reduced with differential GNSS

Fundamentals of GNSS Processing

Appendix A: RELATIVE (DIFFERENTIAL) GNSS

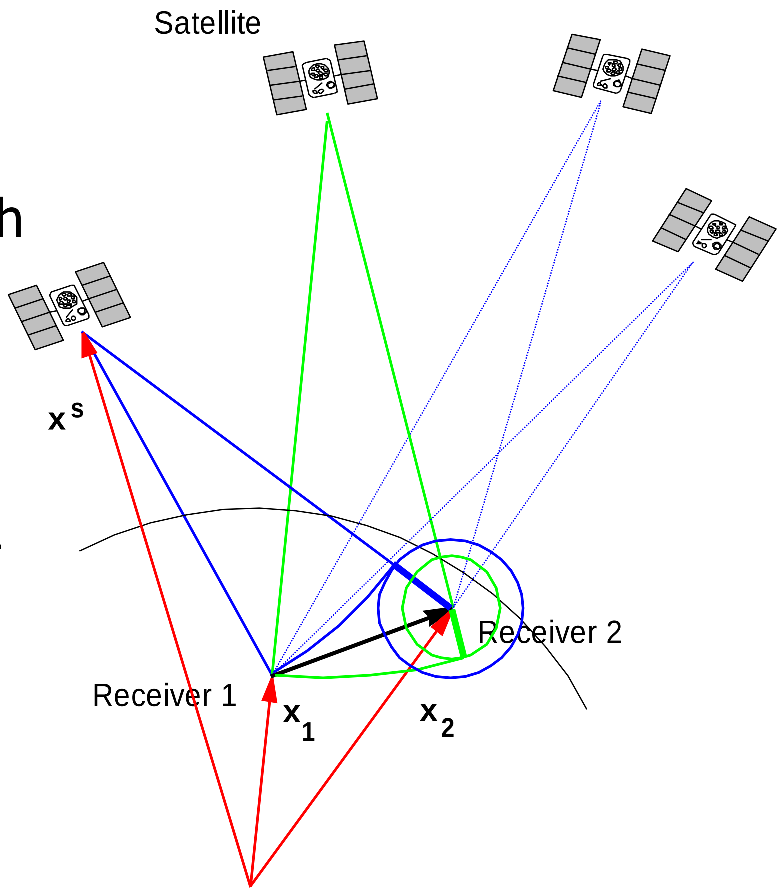
Differential GPS

- Many EO applications require highly accurate positioning measurements
- How?
  By using static solutions if we can
  By using carrier phase measurements
  By using nearby GPS stations whose positions are well known => "Differential GPS" or relative positioning



Differential GPS

- Differential GPS gives accuracy < 1m with code observations and < 2cm with carrier phase
- What happens:
  - GPS orbit error and atmospheric error drop out (distance up to 100km)
  - But noise/multipath increases by factor √2 (two receivers)
  - Carrier phase ambiguities become integers (fixing)



- Differences are formed between the observations of two receivers (as shown) and/or between GPS satellites

Single Difference Carrier Phase

Combinations of 2 observations

single difference between receivers

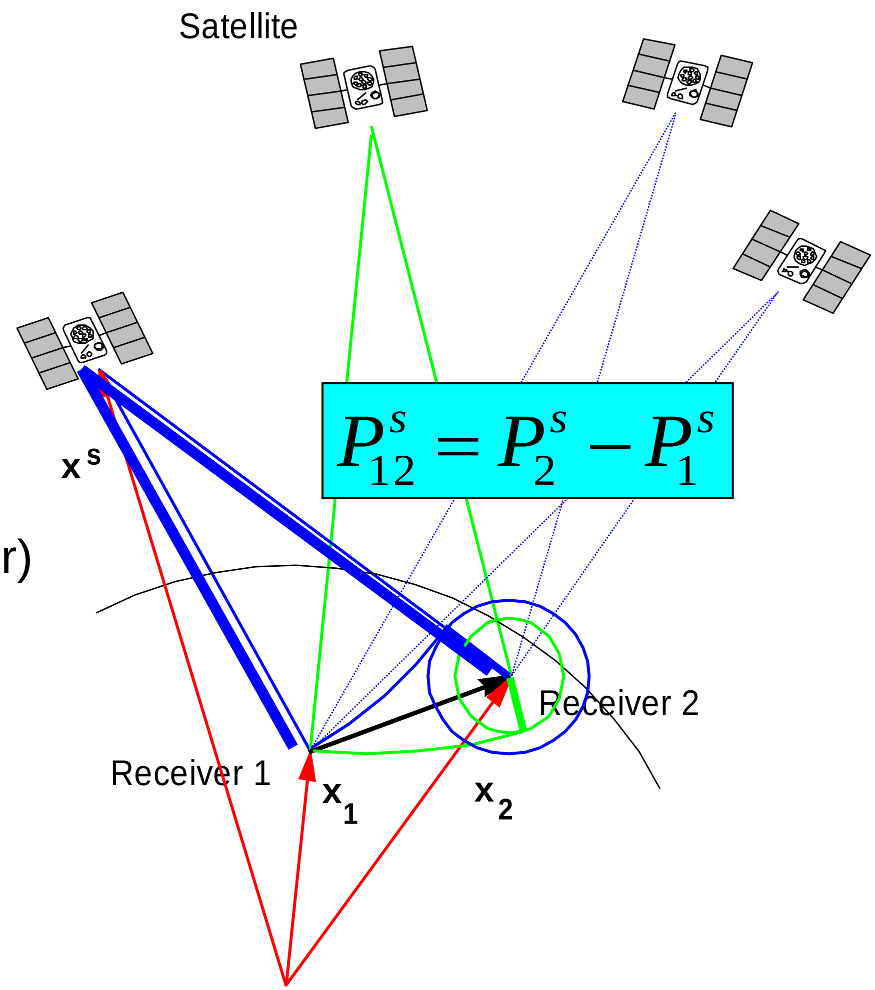
→ Satellite clock error eliminated

→ Unknown parameters

  → Baseline vector

  → Differential receiver clock *)

  → Carrier phase ambiguities *)



*) Model is rank-defect (loose 1 parameter)

  → <u>clock lumped with SD ambiguity</u>

  → double difference ambiguity

Least squares adjustment with

- 3 baseline coordinates
- N receiver clock parameters
- m-1 phase ambiguities

Receivers must be synchronized and track the same satellites (imposes a distance limit)!

Variance of the single difference observation

The variance of the single difference observation is

$$\begin{aligned} D\{\Delta\underline{\bar{P}}_{12}^s\} &= D\{\Delta\underline{P}_{12}^s\} + \mathrm{e}_{12}^{s*}D\{(\Delta\hat{\underline{\mathrm{x}}}^s - \Delta\hat{\underline{\mathrm{x}}}_1)^a\}\mathrm{e}_{12}^s + \gamma_i D\{\Delta\underline{I}_{12}^{s\;a}\} + D\{\Delta\underline{T}_{12}^{s\;a}\} \\ &\simeq 2\sigma_0^2 + \mathrm{e}_{12}^{s*}\mathrm{Q}_{(\Delta\hat{\underline{\mathrm{x}}}^s - \Delta\hat{\underline{\mathrm{x}}}_1)^a}\mathrm{e}_{12}^s + \gamma_i\sigma^2_{\Delta I_{12}^{s\;a}} + \sigma^2_{\Delta T_{12}^{s\;a}} \\ &\simeq 2\sigma_0^2 \end{aligned}$$

orbit & ref.frame *ionosphere* *troposphere*

assuming that the standard deviation of both receivers is identical and are uncorrelated.

The effect of <u>orbits errors</u> is strongly reduced, because

$$\|\mathrm{e}_{12}^s\| = \|\mathrm{e}_2^s - \mathrm{e}_1^s\| < \frac{\|\mathrm{x}_{12}\|}{20000\mathrm{km}} \simeq 0.5\,10^{-8}\|\mathrm{x}_{12}\| \ll 1$$

Only <u>differential atmospheric errors</u> enter the equation.

} Negligible for short baselines

Double Difference Carrier Phase

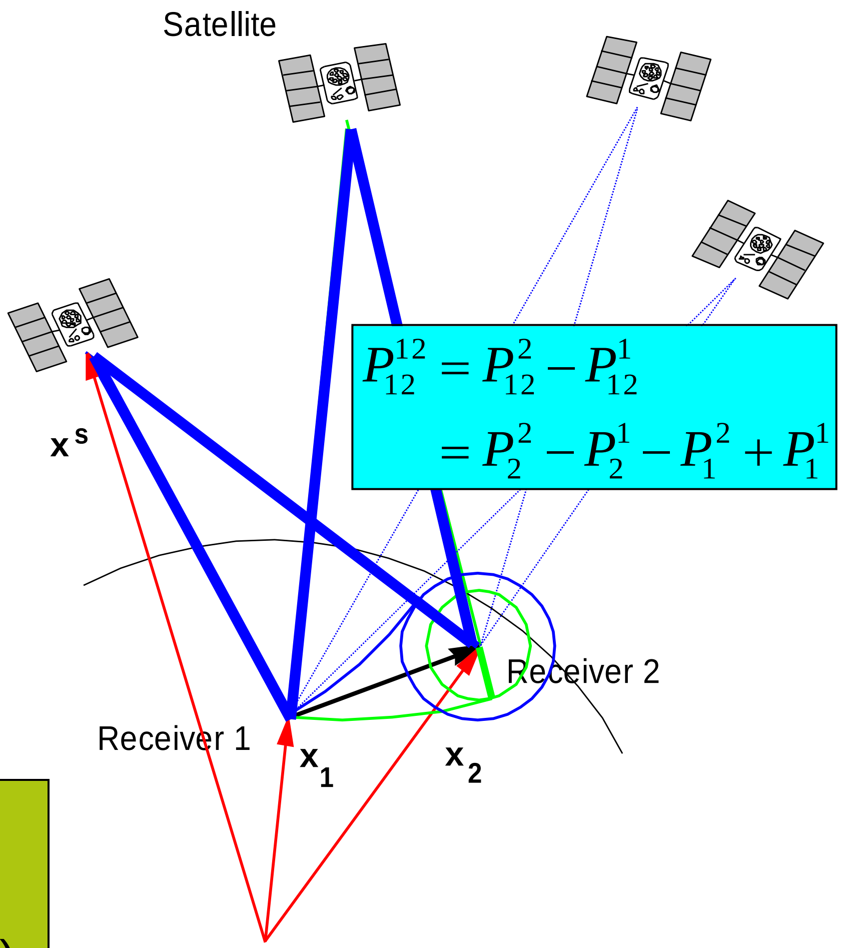
Combinations of 4 observations

single difference between receivers
→ satellite clock error eliminated
single difference between satellites
→ **receiver clock error eliminated**
→ the double difference phase ambiguity becomes an integer number

Least squares adjustment with
- 3 baseline coordinates
- *m* -1 phase ambiguities

m satellites with *m* > 4

RESULTS IDENTICAL TO SINGLE DIFFERENCE!
(if the proper covariance matrix is used)



Double Difference observation equation (1)

- The double difference observation equation for a single epoch is

$$E\{\begin{bmatrix} \Delta\underline{\bar{P}}_{12}^{2} - \Delta\underline{\bar{P}}_{12}^{1} \\ \vdots \\ \Delta\underline{\bar{P}}_{12}^{m} - \Delta\underline{\bar{P}}_{12}^{1} \end{bmatrix}\} = \begin{bmatrix} -(\mathrm{e}_2^2 - \mathrm{e}_2^1)^* & \lambda_i & & \\ \vdots & & \ddots & \\ -(\mathrm{e}_2^m - \mathrm{e}_2^1)^* & & & \lambda_i \end{bmatrix} \begin{bmatrix} \Delta\mathrm{x}_{12} \\ A_{12}^2 - A_{12}^1 \\ \vdots \\ A_{12}^m - A_{12}^1 \end{bmatrix}$$

  we have m_k-1 double difference observations (one less).
- In the double difference equations
  1. The receiver clock error (one per epoch) is eliminated
  2. The rank defect has disappeared

Observations: $(m_k-1)*N$
Unknowns:
- Static: $3 + (m_k-1)$
- Kinematic $3*N + (m_k-1)$

- Easier to solve by least-squares because there are fewer unknowns
- Solution is identical to single difference, as long as the proper double differenced co-variance matrix is used

Double Difference observation equation (2)

- The covariance matrix for the double difference observations is

$$Q_{\Delta \underline{\bar{p}}_{12}^{1S}} = \bar{D} Q_{\Delta \underline{\bar{p}}_{12}^{S}} \bar{D}^* = 2\sigma_0^2 \bar{D}\bar{D}^*$$

$$= 2\sigma_0^2 \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & & 1 \\ \vdots & & \ddots & \\ 1 & 1 & & 2 \end{bmatrix} = 2\sigma_0^2 (I_{m_k - 1} + u_{m_k - 1} u_{m_k - 1}^*)$$

assuming $Q_{\Delta \underline{\bar{p}}_{12}^{S}} = 2\sigma_0^2 I_{m_k}$ is the single–difference co–variance matrix. The double–difference co–variance matrix is not a diagonal matrix, even if the original observations are uncorrelated.

- Drawback of double differences is that the <u>co-variance matrix is not diagonal</u> anymore, as it was the case in the single difference baseline

- However, in case of 3 or more receivers, there will also be significant correlation between otherwise independent single differences.

Difficulties with Carrier Phase Solutions

Using GPS carrier phase measurements have opened the door to a large range of applications; precision of a few millimeters – centimeters.

But…

- Extra phase ambiguities have to be estimated
  - for every satellite receiver pair one ambiguity per frequency
  - constant in time (unless the signal is interrupted)
- Can be computed when satellites move w.r.t. the receiver: i.e. we have to measure over a certain period. The duration depends on
  - the distance to the reference receiver (due to atmospheric effects)
  - number of satellites and the satellite configuration
- Linear combinations (double differences) of phase ambiguities are integer $\Rightarrow$ this property is used to shorten the measurement duration

Ambiguity Resolution

Basis for Real-Time Kinematic (RTK)

- Phase ambiguities can be parameterized as **double differences** in relative model (both in double and single difference models)
- Double-differenced ambiguities should be **integers**:

$$\begin{aligned} A_{12}^{ij} &= A_{12}^{j} - A_{12}^{i} = A_2^j - A_1^j - A_2^i + A_1^i \\ &= N_2^j + \overline{\phi}_2(t_0) - \overline{\phi}^j(t_0) - N_1^j - \overline{\phi}_1(t_0) + \overline{\phi}^j(t_0) - \\ &\quad N_2^i - \overline{\phi}_2(t_0) + \overline{\phi}^i(t_0) + N_1^i + \overline{\phi}_1(t_0) - \overline{\phi}^i(t_0) \\ &= N_2^j - N_1^j - N_2^i + N_1^i \end{aligned}$$

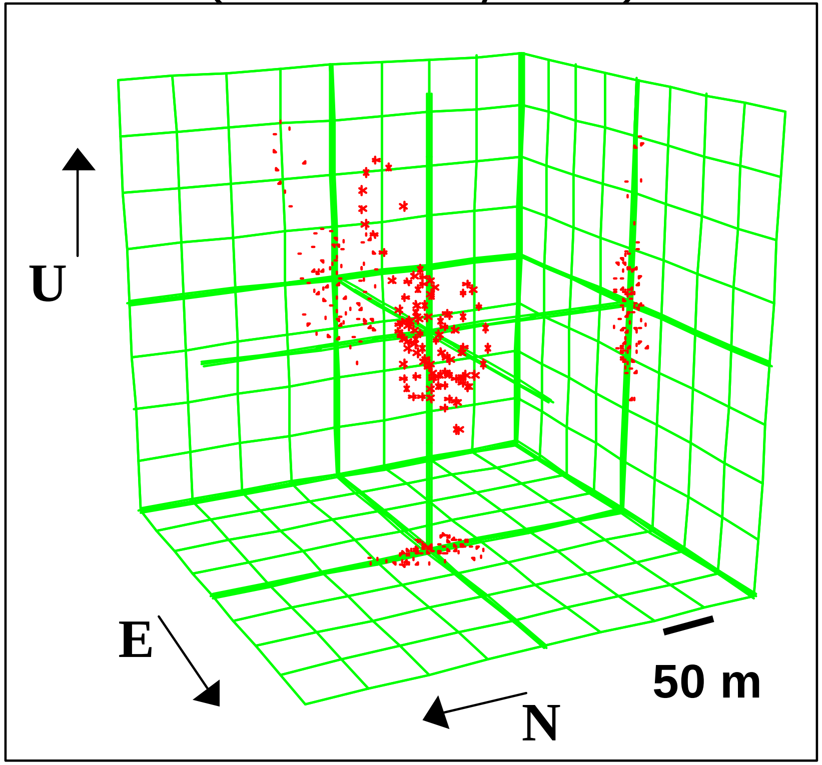
where $A \in R$ and $N \in Z$

- When integer ambiguities are **resolved** and **fixed**, phase data start to act as code data (pseudoranges), but with very high precision!

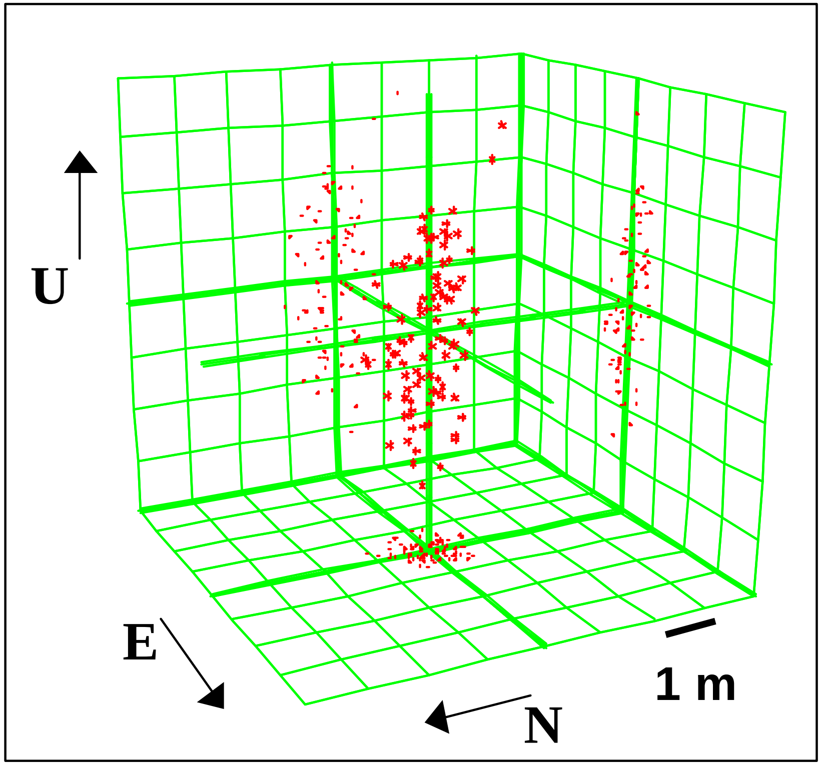
$\Rightarrow$ **short observation times** may be used

Impact of Differential GPS

SA is on (before 2 May 2000)



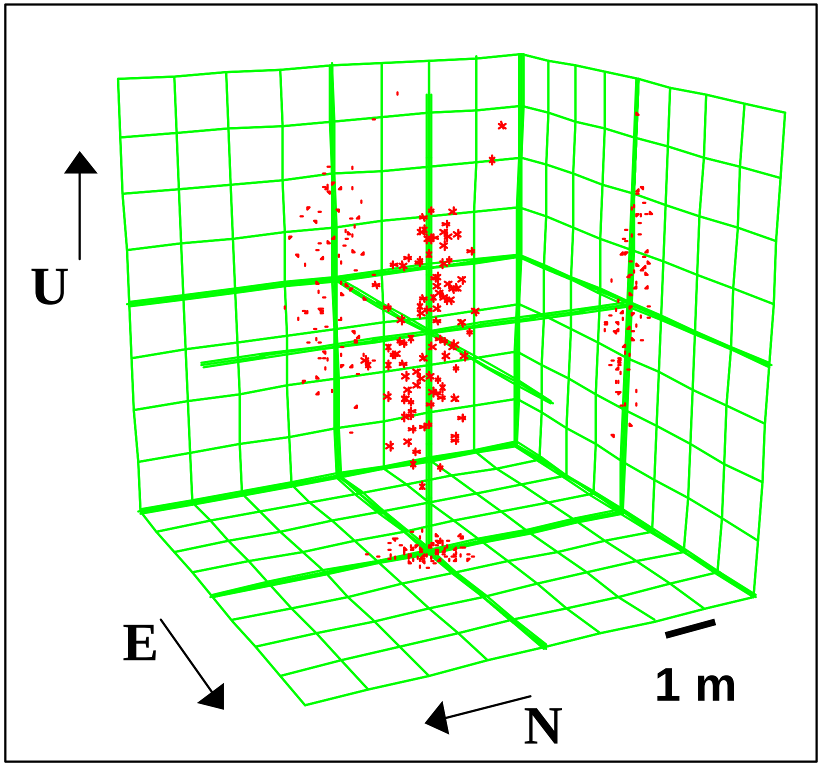
Single Point Positioning with pseudo ranges (code)



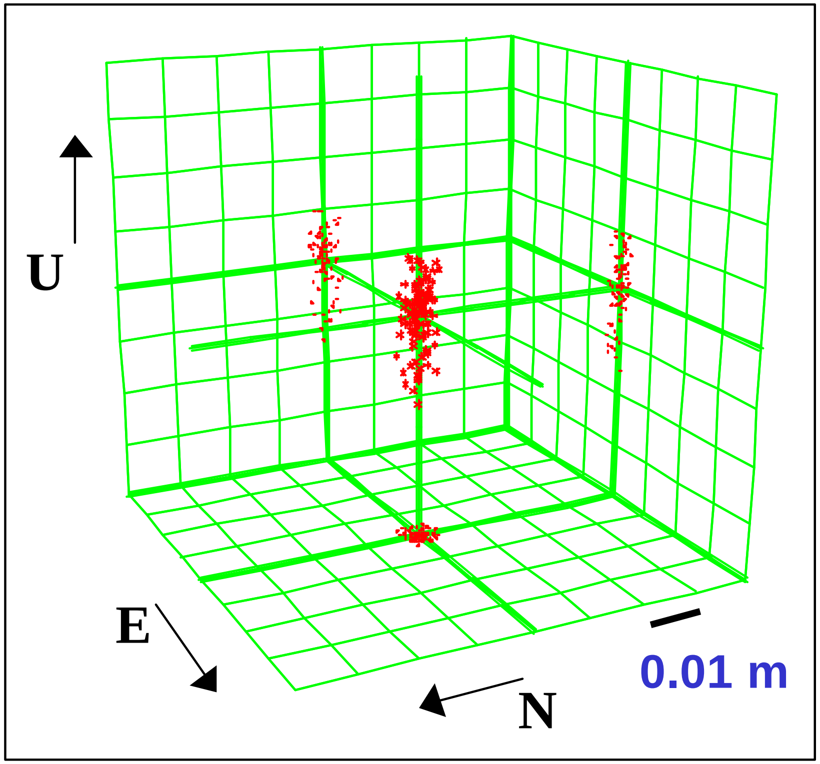
Relative positioning with pseudo ranges (code)

Impact of Carrier Phase Measurements

With relative positioning (DGPS)



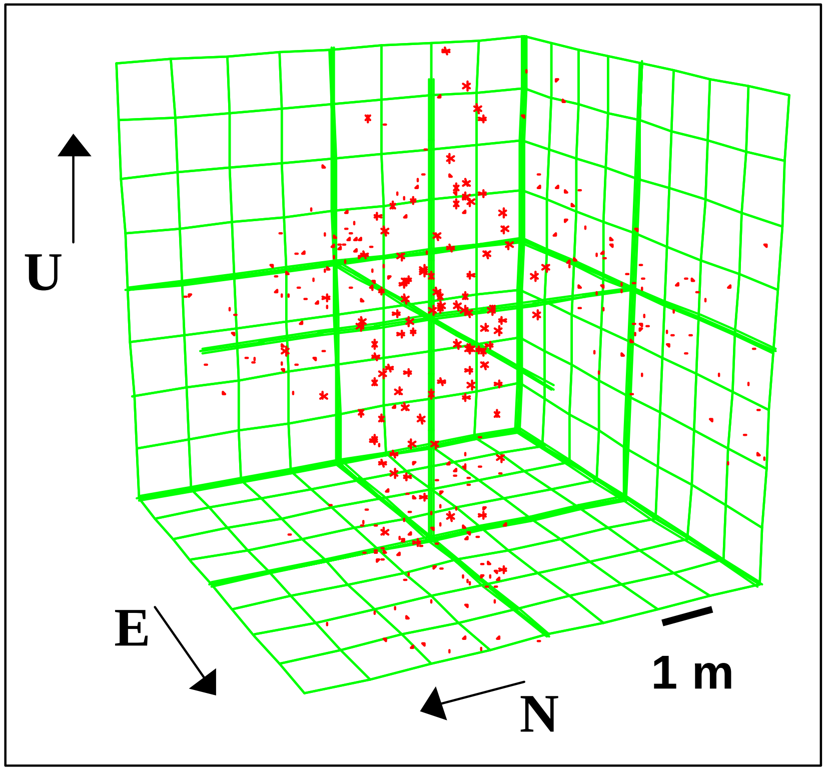
Relative positioning with pseudo ranges (code)



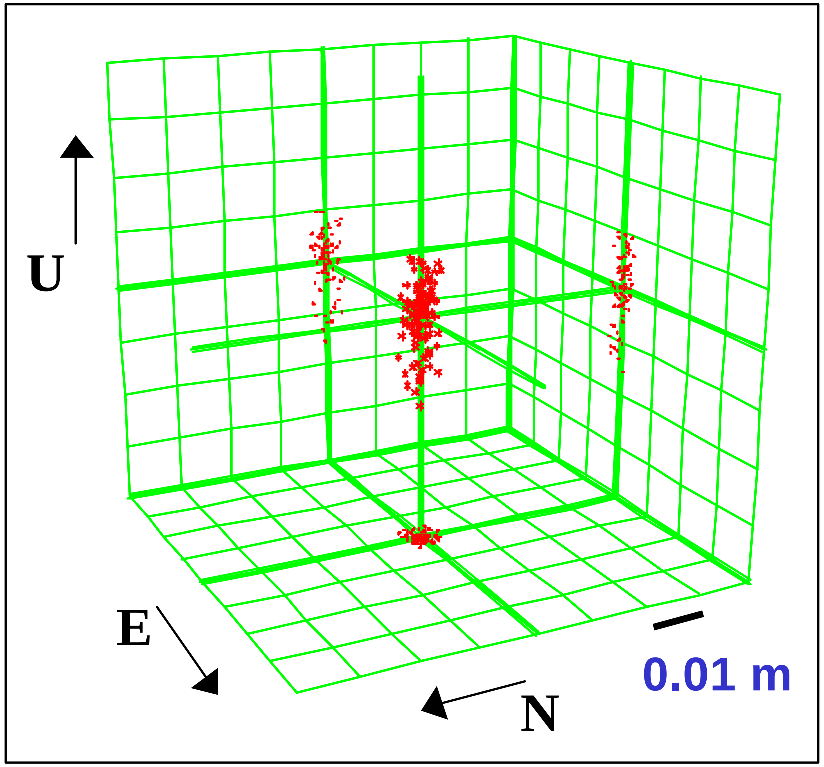
Relative positioning with carrier phase data (ambiguities resolved: **fixed**)

Impact of Ambiguity Resolution (AR)

Even worse than code data -> depends on measurement interval (extreme example)



Relative positioning with carrier phase data (ambiguities estimated: **float**)



Relative positioning with carrier phase data (ambiguities resolved: **fixed**)

Baseline precision

For single and double difference solutions!

- The co-variance matrix of the baseline vector (3 coordinates) in case of carrier phase data is (ambiguity float)

$$2\sigma_0^2\left(\sum_{k=0}^{N}(\mathrm{e}_{2\,k}^S - \bar{\mathrm{e}}_2^S)(\mathrm{I}_{m_k} - \frac{1}{m_k}\mathrm{u}_{m_k}\mathrm{u}_{m_k}^*)(\mathrm{e}_{2\,k}^S - \bar{\mathrm{e}}_2^S)^*\right)^{-1}$$

between-receiver-differences (factor 2)

between-satellite-differences to eliminate Rx clock

Change in satellite geometry (due to float ambiguities (carrier phase)

- In case of pseudo range data / carrier phase ambiguity fixed

$$2\sigma_0^2\left(\sum_{k=0}^{N}\mathrm{e}_{2\,k}^S(\mathrm{I}_{m_k} - \frac{1}{m_k}\mathrm{u}_{m_k}\mathrm{u}_{m_k}^*)\mathrm{e}_{2\,k}^{S*}\right)^{-1}$$

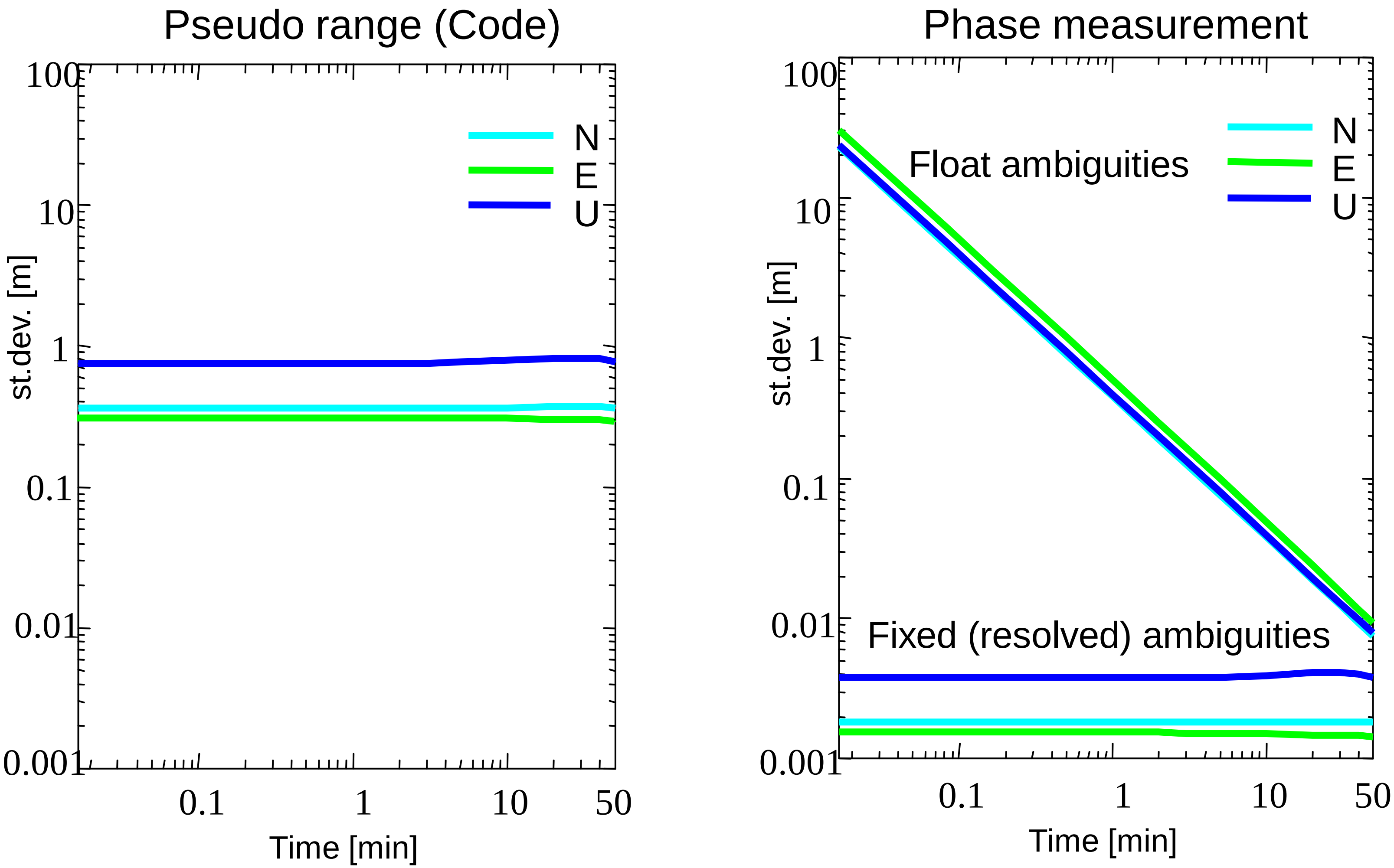
$\sigma_0(\text{code}) \cong .30 - 1\, m$

$\sigma_0(\text{phase}) \cong 1 - 2\, mm$

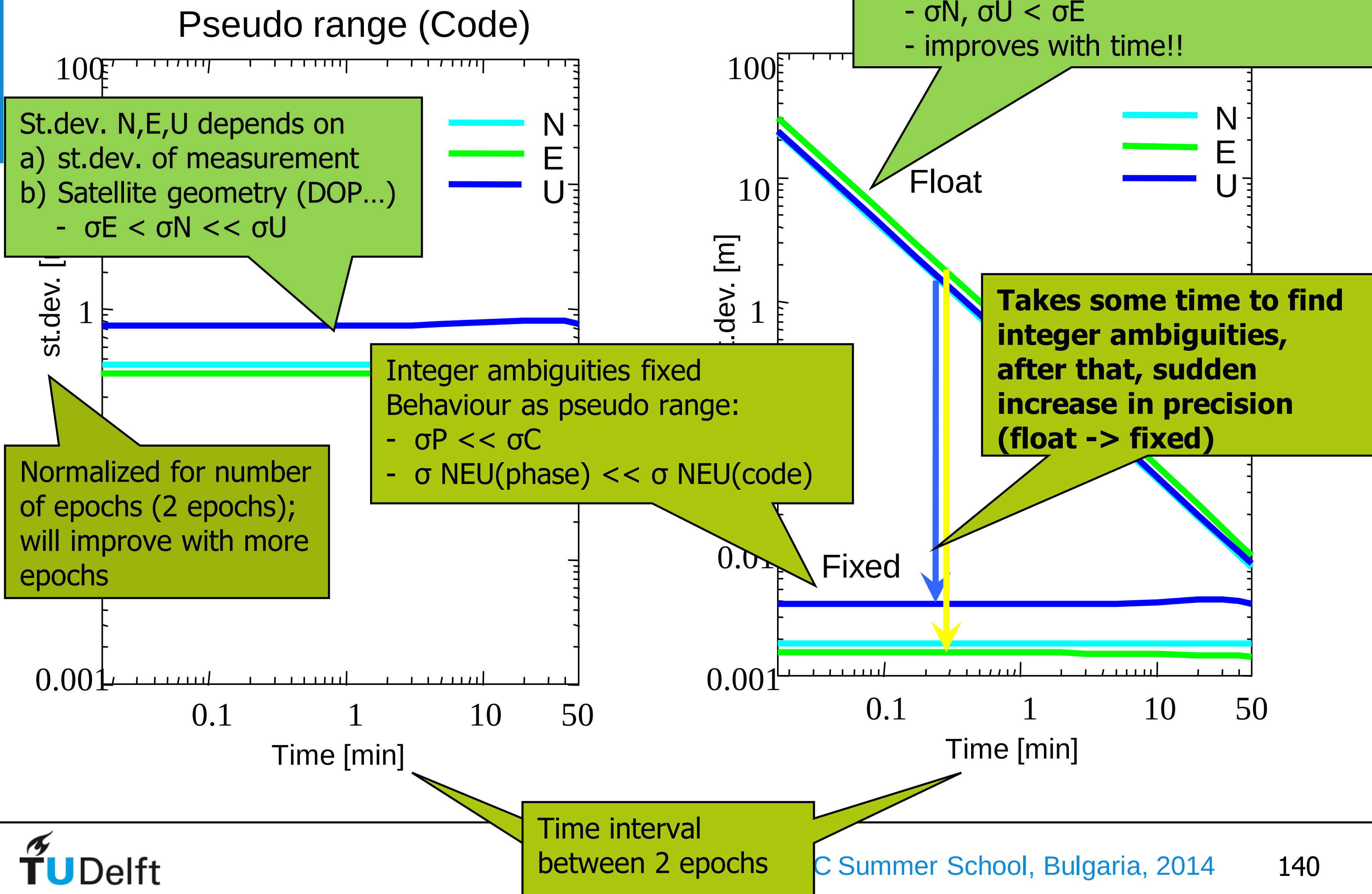
Satellite geometry

- In case of carrier phase data (with float ambiguities) the precision depends also on the **<u>change in satellite geometry</u>**

Impact of AR on baseline precision

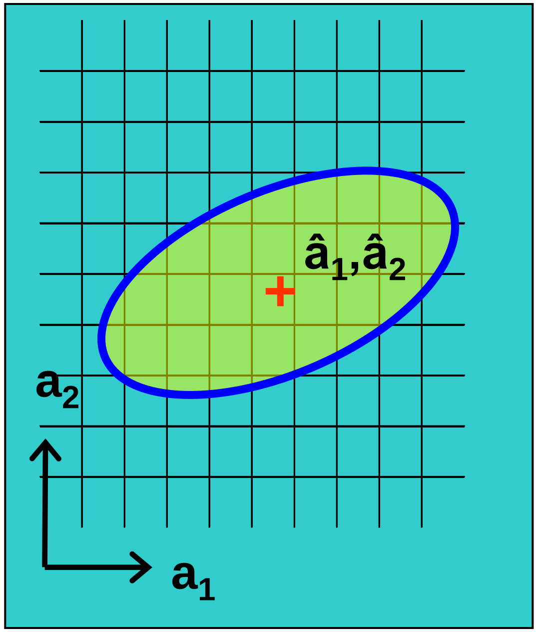


Impact of AR …


Pseudo range (Code)
100
1
0.001
st.dev. [
0.1
1
10
50
Time [min]
N
E
U
St.dev. N,E,U depends on
a) st.dev. of measurement
b) Satellite geometry (DOP…)
- σE < σN << σU
Normalized for number of epochs (2 epochs); will improve with more epochs
Integer ambiguities fixed
Behaviour as pseudo range:
- σP << σC
- σ NEU(phase) << σ NEU(code)
St.dev. N,E,U depends on
a) st.dev. of measurement: σP << σC
b) Change in satellite geometry
- σN, σU < σE
- improves with time!!
100
10
1
0.001
.dev. [m]
Float
Fixed
Takes some time to find integer ambiguities, after that, sudden increase in precision (float -> fixed)
Time [min]
Time interval between 2 epochs

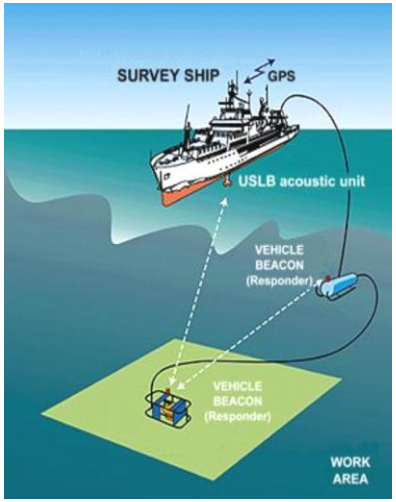
Applications of Ambiguity Resolution

Differential + Use of carrier phase (mm accuracy, but ambiguous)



surveying

machine guidance



sea-floor mapping

LAMBDA method: fast and efficient GNSS carrier phase cycle ambiguity resolution (made in Delft)
⇒<u>real-time precise surveying</u>

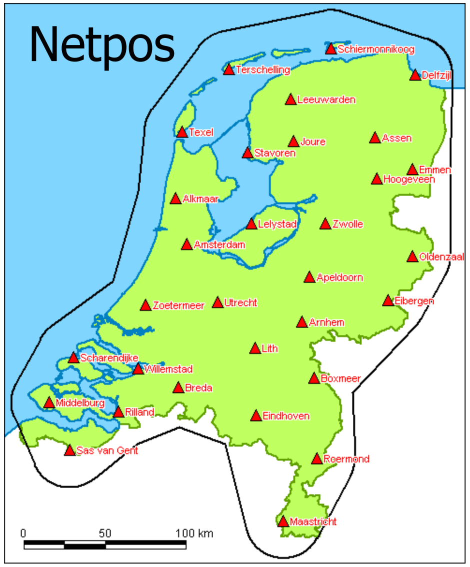
Development of models and algorithms for data processing
⇒ high precision applications

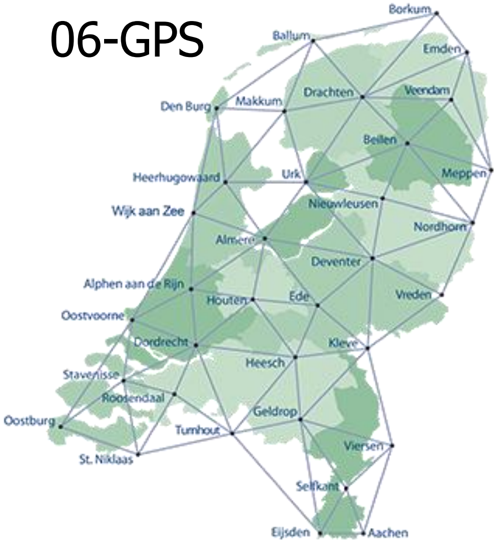
Network RTK

- RTK ("Real Time Kinematic")
  - Centimeter accuracy through the use of carrier phase and differential processing (baselines); ambiguity resolution is key.
  - Baseline length limited to 20-30 km (need nearby reference station)
- Network RTK
  - Network instead of single baseline -> improved integrity, availability
  - Interpolation of atmospheric delays -> larger distances between RTK stations (50-70 km)
  - Network RTK implementations
    - Virtual GPS reference stations; more centralized processing
    - Master-auxiliary concept (MAC or MAX) and Flachen-Korrektur-Parameter (FKP); efficient packing of data of multiple reference stations, processing by user receiver
- Wide Area RTK
  - Network station spacing >> 100 kilometers
  - Main issue: ionosphere corrections

Network RTK

Examples of RTK networks in the Netherlands





Average distance between stations in each network is 40km

Data available in real-time via GPRS (NTRIP) for surveyors –>

hence the name "real-time kinematic" **RTK**

NETPOS is also made available for meteorological applications

It is time to put IGS in the spotlight…

Fundamentals of GNSS Processing

Appendix B: INTERNATIONAL GNSS SERVICE (IGS)

International GNSS Service (IGS)

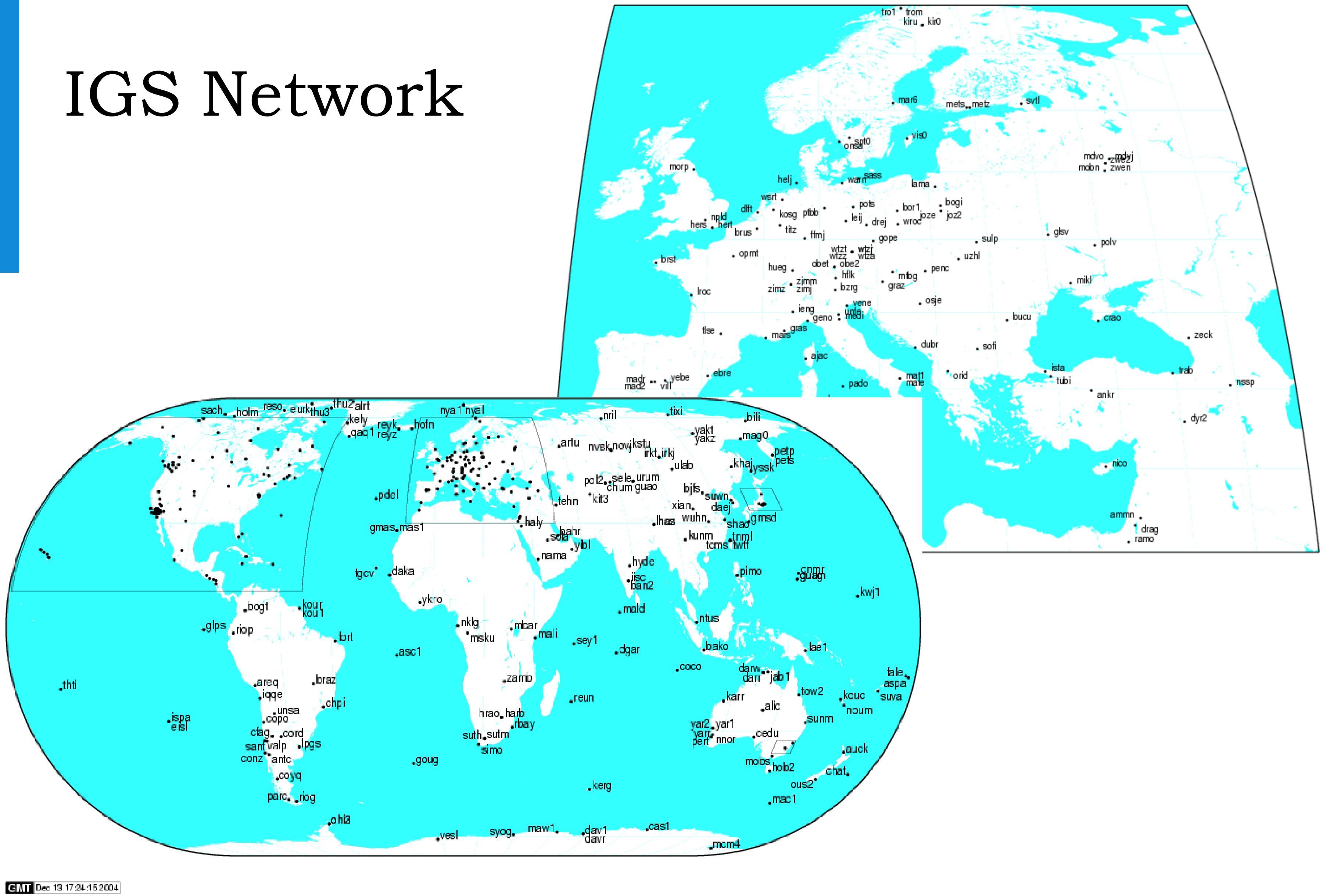
Network of more than 200 GNSS stations, 5 regional and 3 global data centers, 9 analysis centers and a central bureau

IGS products

- GNSS satellite orbits and clocks
  - Final and Rapid products for post-processing
  - Ultra rapid products for (near-) realtime processing
- Earth rotation parameters
- IGS stations coordinates and velocities
- Atmosphere parameters (Total Electron Content, Water Vapour)
- Tracking data of the IGS stations

Includes GPS and GLONASS, and other systems on experimental basis

IGS Network



sach
holm
reso
eurk
thu3
thu2
alrt
kely
qaq1
reyk
reyz
hofn
nya1
nyal
nril
tixi
bili
yakt
yakz
mag0
artu
nvsk
novj
kstu
irkt
irkj
petp
pets
khaj
yssk
ulab
pdel
pol2
sele
urum
chum
guao
kit3
bjfs
suwn
daej
tehn
xian
wuhn
lhas
shao
gmsd
gmas
mas1
haly
bahr
sola
kunm
tnml
tcms
twtf
yibl
nama
hyde
iisc
ban2
pimo
cnmr
guam
kwj1
tgcv
daka
ykro
bogt
kour
kou1
glps
riop
fort
nklg
msku
mbar
mali
sey1
mald
ntus
bako
lae1
asc1
dgar
coco
darw
darr
jab1
fale
aspa
suva
thti
areq
iqqe
braz
chpi
unsa
copo
ispa
eisl
ctag
cord
lpgs
valp
conz
antc
zamb
reun
hrao
harb
rbay
suth
sutm
simo
karr
alic
tow2
kouc
noum
sunm
yar2
yar1
nnor
pert
cedu
auck
goug
mobs
hob2
chat
coyq
parc
riog
kerg
ous2
mac1
ohi2
vesl
syog
maw1
dav1
davr
cas1
mcm4
tro1
trom
kiru
kir0
mar6
mets
metz
svtl
onsa
vis0
mdvo
mdvj
mobn
zwen
morp
helj
warn
sass
lama
wsrt
dlft
kosg
ptbb
pots
bor1
bogi
hers
nrld
hert
brus
tit
ffmj
leij
drej
wroc
joze
joz2
gope
sulp
glsv
polv
brst
opmt
wtzt
wtzj
wtzz
wtza
uzhl
hueg
obet
obe2
hflk
mkog
penc
zimz
zimm
zimj
bzrg
graz
mikl
lroc
ieng
geno
vene
upad
medi
osje
bucu
crao
tlse
mars
gras
dubr
sofi
zeck
ajac
ista
tubi
trab
nssp
madr
mad2
vill
yebe
ebre
pado
mat1
mate
orid
ankr
dyr2
nico
ammn
drag
ramo
GMT Dec 13 17:24:15 2004

Accuracy of IGS Products

	Final[1)] (2 weeks)	Rapid (2 days)	Ultra-rapid[2)] (real time)
Satellite orbit	< 5 cm	5 cm	10-15 cm
Satellite clock	0.1 ns	0.2 ns	5 ns
Pole	0.1 mas	0.2 mas	--
Polar motion	0.2 mas/day	0.4 mas/day	--
Length of day	20 us/day	20 us/day	--

1) Including daily solutions of station coordinates (in SINEX format) within 2-3 weeks (3 mm - 5 mm)

2) Twice per day, with prediction for next 24 hours

IGS Orbit and Clock Products

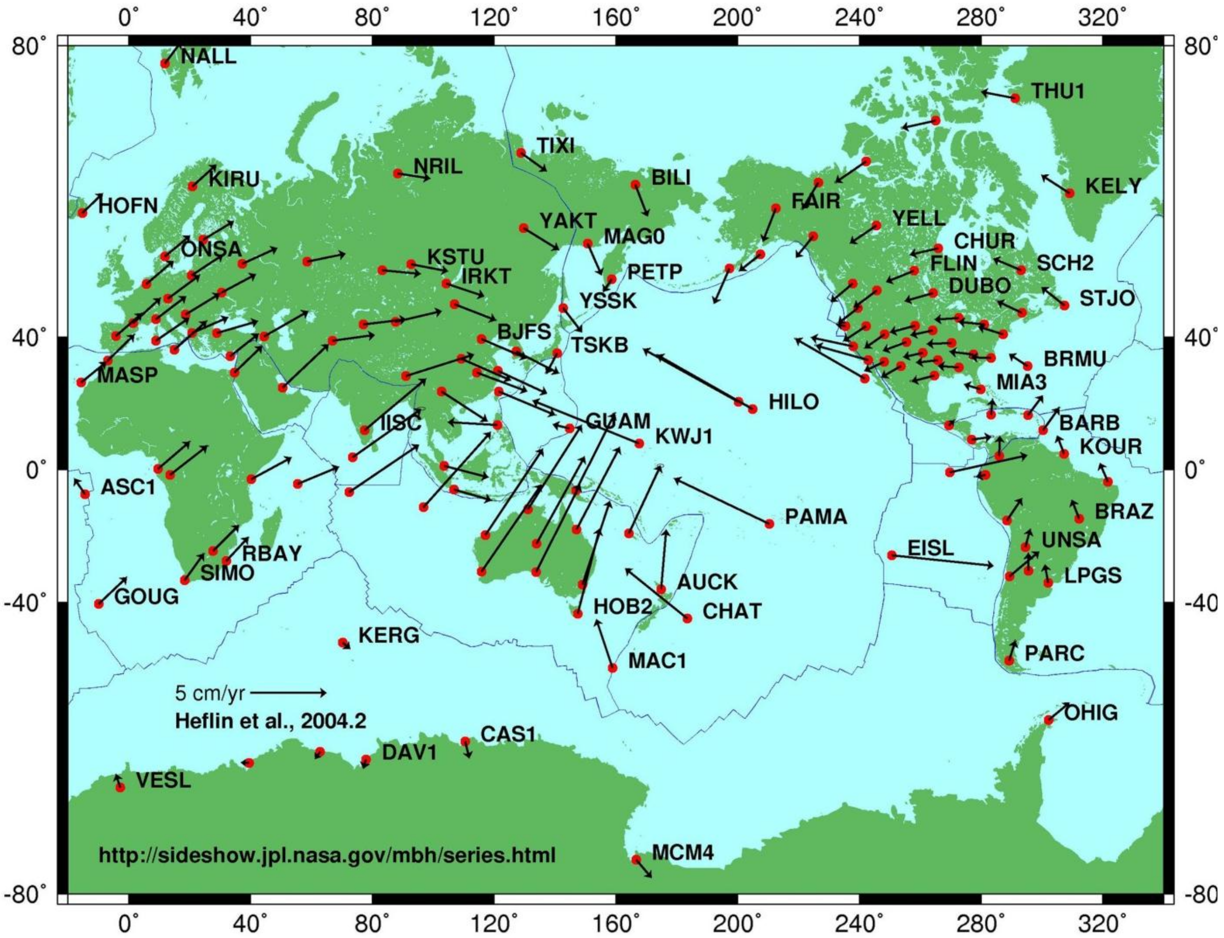
GPS satellite orbits and clocks are the main IGS product:

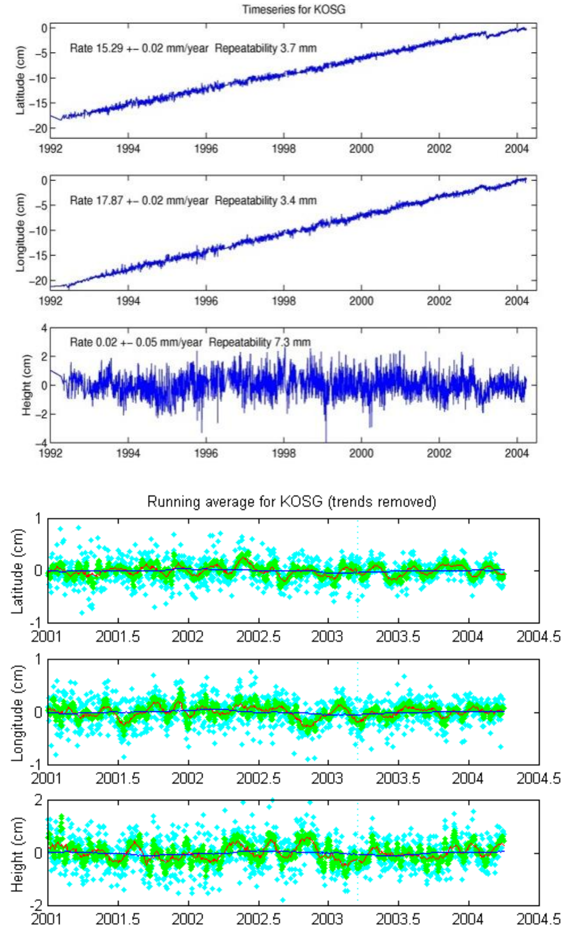
	Accuracy	Delay	Frequency	Data Interval
Broadcast	~260 cm/~7 ns	real time	2 hour	--
Ultra-Rapid	~25 cm/~5 ns	real time	2x day	15 min/5 min
Rapid	~5 cm/0.2 ns	17 hour	daily	15 min/5 min
Final	<5 cm/0.1 ns	~13 days	weekly	15 min/5 min

IGS orbits and clocks are an excellent replacement for the broadcast orbits
➔ centimeter-decimeter accuracy

IGS orbits and clocks are correlated and must be used together
➔ **Precise Point Positioning**

IGS Postions and Velocities





Fundamentals of GNSS Processing

Appendix C: TROPOSHERE AND MAPPING FUNCTIONS

Composition of the troposphere

The troposphere contains both dry air and water vapour:

- Dry air has no significant variation in composition with latitude and height.

- The amount of water vapour, on the other hand, varies widely, both spatially and temporally. Most of the water vapour is contained in the boundary layer, the lowest 2 km of the troposphere.

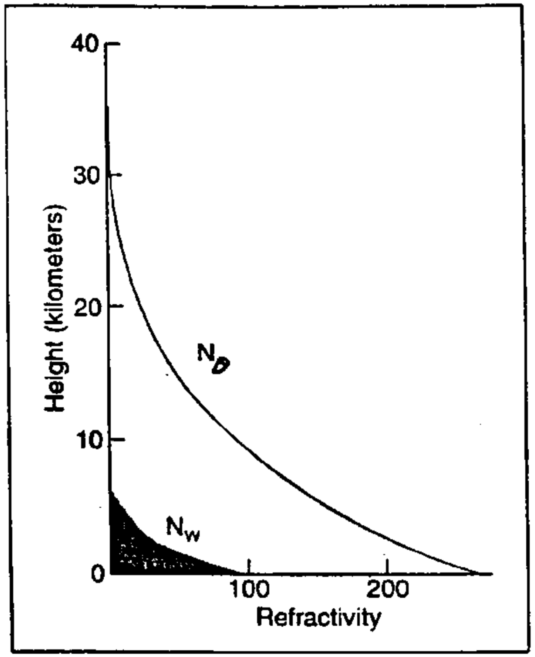
The mixture is called moist air.

Moist air behaves (almost) like an ideal gas.

Troposphere delay and Refractivity

The tropospheric delay (better: neutral atmospheric delay) is given by

$$\Delta\rho = 10^{-6}\int_A^B N ds = 10^{-6}\int_A^B N_d\, ds + 10^{-6}\int_A^B N_w\, ds = \Delta\rho_d + \Delta\rho_w$$



The integral equation can be evaluated using

- the actual refractivity profile, or
- it may be approximated by an analytical function, based an a simple model.

Profiles of N_w show strong variations with time and location, and are generally difficult to predict.

The variability is largest in the regions of

- 0 km - 1.5 km (atmospheric boundary layer)

- cloud layers (usually below 4 km)

The wet part is only significant in the lower part of the troposphere.

The variability of N_d is very small because of the constant ratio of constituents

Refractivity of air (1)

- The refractivity N of the atmosphere can be split into refractivity of dry are N_d and refractivity of water vapour N_v

$$N = N_d + N_v$$

The dry and wet refractivity of a parcel of air are given by the following model [Thayer, 1994]

$$N_d \doteq k_1 \frac{P_d}{T} Z_d^{-1} = k_1 \; R_d \rho_d \cong k_1 \frac{P_d}{T}$$

$$N_v \doteq [k_2 \frac{e}{T} + k_3 \frac{e}{T^2}] Z_v^{-1} = (k_2 + k_3/T) R_v \rho_v \cong k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

The compressibility factors Z_d and Z_v are close to one.

with P_d and e the partial pressures of dry air and water vapour $[hPa]$ $(1\,hPa = 1\,mbar = 100 Nm^{-2})$ and T the temperature $[K]$. The values for the coefficients k_1, k_2 and k_3 are empirically derived.

Note: $\frac{P_i}{T} Z_i^{-1} = \rho_i R_i$ with ρ the density and R the specific gas constant (equation of state)

The accuracy of N_d is estimated conservatively at 0.02% of N_d. A conservative

Refractivity of air (2)

The dry refractivity and first term of the water vapour refractivity are the result of induced molecular polarization.

The second, and much larger term of water vapour refractivity, represents the effect of the permanent dipole moment of the water vapour molecule.

The figure shows the contribution of both parts of the wet refractivity for saturation pressures.

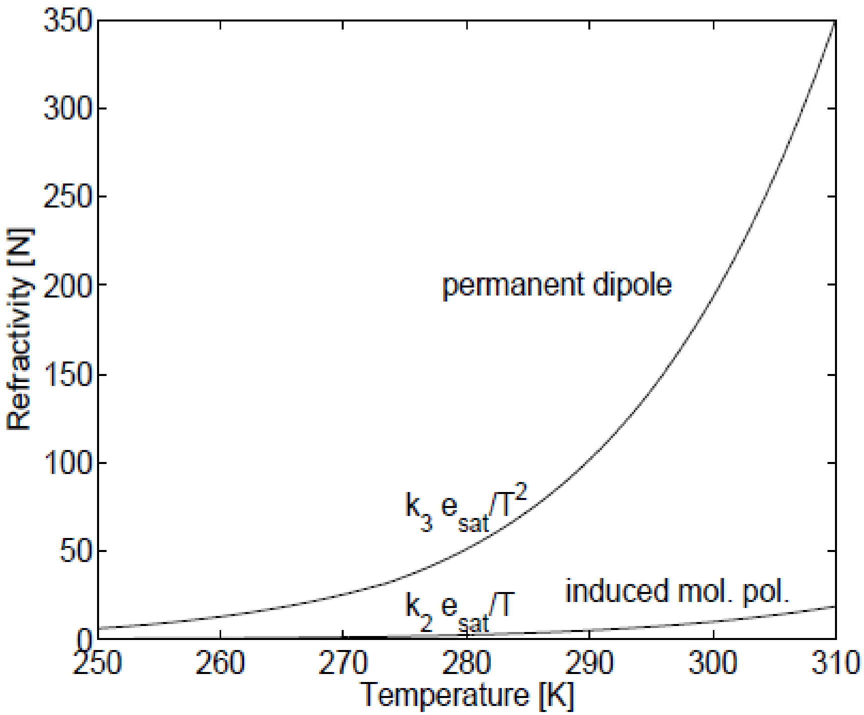


Figure courtesy of Frank Kleijer, 2004.

Refractivity of air (3)

- Instead of splitting refractivity into a dry and water vapour part, we can also split it into a hydrostatic N_h and non-hydrostatic N_w part

$$N = N_h + N_w$$

The hydrostatic and non-hydrostatic refractivity are given by

$$N_h \doteq k_1 \frac{P}{T_v} Z_d^{-1} = k_1 R_d \rho_m \cong k_1 \frac{P}{T}$$

The compressibility factors Z_d and Z_v are close to one; $T_v \cong T$.

$$N_w \doteq [k'_2 \frac{e}{T} + k_3 \frac{e}{T^2}] Z_v^{-1} = (k'_2 + k_3/T) R_v \rho_v \cong k'_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

with P the total pressure $[hPa]$ and e the partial pressure of water vapour $[hPa]$ ($1\ hPa = 1\ mbar = 100 Nm^{-2}$), T the temperature $[K]$ and

$$k'_2 = k_2 - k_1 \frac{R_d}{R_v} = k_2 - k_1 \epsilon$$

Empirical values for the constants k_i

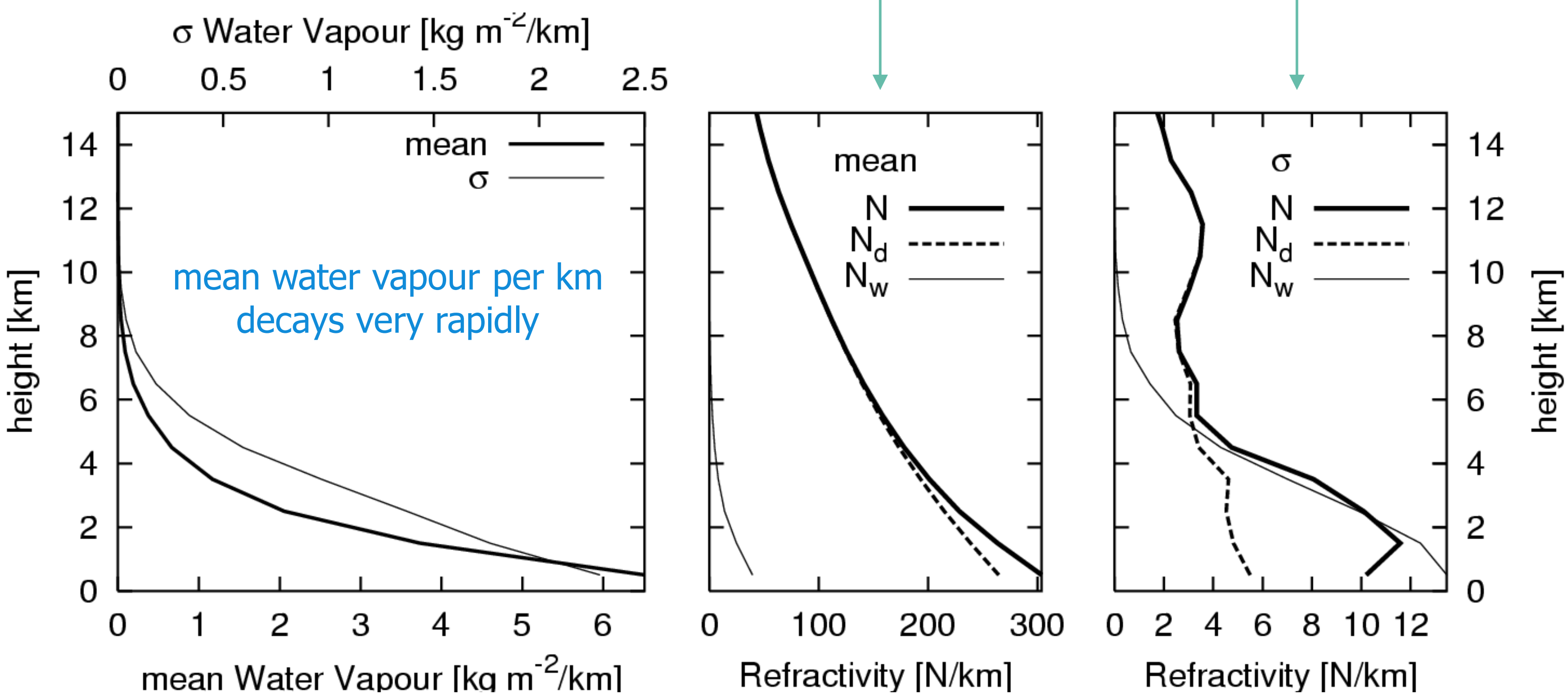
Author(s)	k_1 $[K\ hPa^{-1}]$	k_2 $[K\ hPa^{-1}]$	k_3 $[10^5\ K^2 hPa^{-1}]$
Essen & Froome, 1951 IUGG standards 1963	77.624	64.7	3.71897
Smith & Weintraub, 1953	77.607± 0.013	71.6± 8.5	3.747± 0.013
Boudouris, 1963	77.59 ± 0.08	72 ± 11	3.75 ± 0.03
Thayer, 1974	77.604± 0.014	64.79± 0.08	3.776±0.004
Hasegawa & Strokesburry, 1975	77.6±0.032	69.4± 0.146	3.701± 0.003
Liebe, 1996	77.676± 0.023	71.631	3.74656
Bevis et al., 1994 weighted	77.6	69.4	3.701
Bevis et al., 1994 unweighted	77.6± 0.05	70.4±2.2	3.739± 0.012
Rüeger, 2002 best available	77.695± 0.013	71.97± 10.5	3.75406± 0.03
Rüeger, 2002 best average	77.689±0.0094	71.295± 1.3	3.75463± 0.0076

(Values for air with 0.0375% (375 ppm) content of CO_2).

Annual mean and variation of water vapour

wet refractivity is more variable than the dry in the lowest 4 km

Above 4 km the refractivity is dominated by the dry refractivity



Annual mean (thick) and variation (thin) of water vapour

Annual mean and variation of total (thick), dry (dashed) and wet (thin) refractivity.

Figure courtesy of Siebren de Haan, 2008.

Annual mean and variation of water vapour

- The maximum amount of water vapour that can persist in a volume of air is dependent on the air temperature. Colder air can contain less water vapour than warmer air.
- The mean water vapour per km decays very rapidly and attains zero at an height of approximately 10 km. The top of the tropopause is very dry (with respect to water vapour) due to temperatures of around 225 K.
- The spread in water vapour decays also very rapidly from values of 2.5 at the surface to zero at 10 km.
- Effect on refractivity (important for GPS):
  - Above 4 km the refractivity is dominated by the dry refractivity. The wet refractivity is 10% of the total refractivity.
  - The spread of the refractivity shows that the wet refractivity is more variable than the dry in the lowest 4 km. Moreover, both quantities are not independent, due to the dependence on temperature, as can be seen by comparing the spread of dry and wet refractivity.

Zenith Hydrostatic Delay (ZHD)

- The ZHD can be written as

$$ZHD \doteq 10^{-6}\int_{h_0}^{\infty} N_h(h)dh = 10^{-6}k_1R_d\int_{h_0}^{\infty}\rho_m(h)dh$$

Under the condition of hydrostatic equilibrium (Appendix I) we have

$$\frac{dP}{dh} = -\rho_m(h)g(h)$$

where $g(h)$ is the acceleration due to gravity. Integration yields

$$\int_{P_0}^{0} dP = \int_{h_0}^{\infty} -\rho_m(h)g(h)dh = -P_0$$

with P_0 the surface pressure $[hPa]$ at height h_0. The ZHD now becomes

$$ZHD = 10^{-6}\frac{k_1R_d}{g_m}P_0 = 10^{-6}\frac{k_1R_d}{g_m^0}\frac{P_0}{f(\phi,h_0)} \cong 0.0022768\frac{P_0}{f(\phi,h_0)}$$

with $g_m \doteq g_m^0.f(\phi,h_0)$ the mean gravity, $g_m^0 = 9.784\ ms^{-2}$
and $f(\phi,h_0) = 1 - 2.66\ 10^{-3}cos2\phi - 2.8\ 10^{-7}\ h \approx 1.$

$$g_m \doteq \frac{\int_{h_o}^{\infty}\rho_m(h)g(h)dh}{\int_{h_o}^{\infty}\rho_m(h)dh}$$

Zenith Wet Delay (ZWD) – IWV model

- The ZWD can be written as

$$ZWD \doteq 10^{-6} \int_{h_0}^{\infty} N_w(h) dh = 10^{-6}(k_2' + k_3/T_m) R_v \int_{h_0}^{\infty} \rho_v(h) dh =$$

$$10^{-6}(k_2' + k_3/T_m) \int_{h_0}^{\infty} \frac{e(h)}{T(h)} dh = Q \cdot IPWV$$

with T_m a mean temperature of water vapour defined as

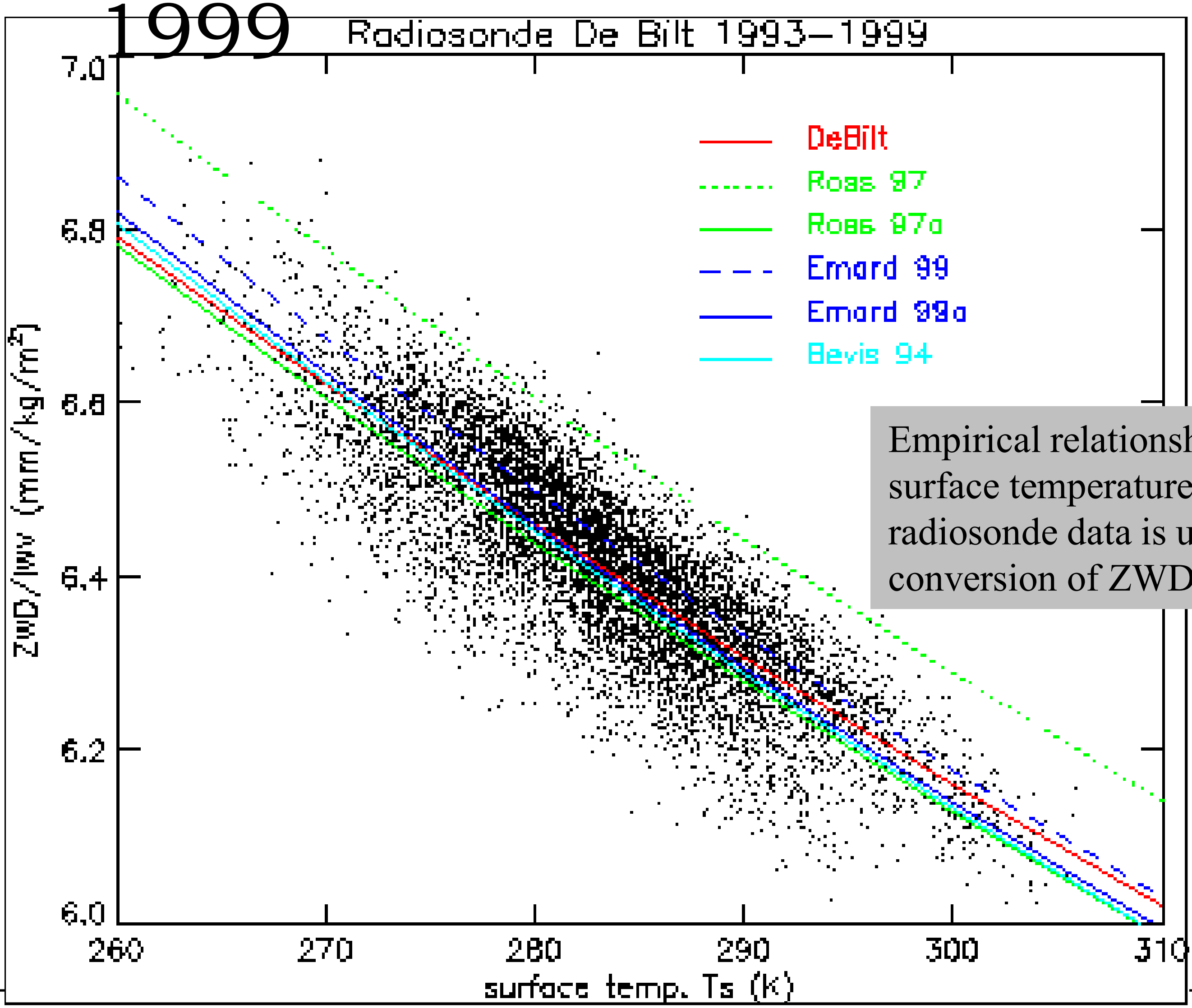
$$T_m \doteq \frac{\int_{h_o}^{\infty} \rho_v(h)\, dh}{\int_{h_o}^{\infty} \rho_v(h)/T(h) dh} = \frac{\int_{h_o}^{\infty} e(h)/T(h)\, dh}{\int_{h_o}^{\infty} e(h)/T(h)^2 dh} \approx 70.2 + 0.72\, T_0$$

The Integrated Water Vapour (IWV) and precipital water vapour (IPWV) are defined as

$$IWV \doteq \int_{h_0}^{\infty} \rho_v(h) dh \qquad [kg\, m^{-2}] \qquad\qquad IPWV \doteq IWV/\rho_w \qquad [m]$$

with $\rho_w = 1000\, kg\, m^{-2}$ the density of water and $Q = 10^{-6}(k_2' + k_3/T_m) R_v \rho_w$. The factor $Q \approx 6.5$ is dimensionless and varies spatially and temporally.

$Q(T_s)$ from radiosonde De Bilt 1993-1999



Empirical relationship $Q(T_s)$ with surface temperature; line fitted to radiosonde data is used in the conversion of ZWD to IWV

Zenith Total Delay - Summary

- The Zenith Total Delay (ZTD) is the sum of ZHD and ZWD

$$ZTD \doteq 10^{-6} \int_{h_0}^{\infty} [N_h(h) + N_w(h)] dh = ZHD + ZWD$$

The **ZHD** is a function of the **pressure** P_0 $[hPa]$ at **height** h_0 and **latitude** ϕ

$$ZHD = 10^{-6} \frac{k_1 R_d}{g_m} P_0 = 10^{-6} \frac{k_1 R_d}{g_m^0} \frac{P_0}{f(\phi, h_0)} \cong 0.0022768 \frac{P_0}{f(\phi, h_0)}$$

with $g_m \doteq g_m^0 . f(\phi, h_0)$, $f(\phi, h_0) = 1 - 2.66\, 10^{-3} cos2\phi - 2.8\, 10^{-7}\, h_0 \approx 1$, $g_m^0 = 9.784\, ms^{-2}$, $R_d = 287.06\, m^2 s^{-2} K^{-1}$ and $k_1 = 77.604\, K\, hPa^{-1}$. The accuracy is $0.5 - 2\, mm/bar$ depending on the values used for k_1 and R_d (which may differ slightly between various models), but assuming zero error in P_0.

The **ZWD** is a function of **IWV** and **mean temperature** T_m

$$ZWD = 10^{-6}(k_2' + k_3/T_m) R_v\, IWV \cong 10^{-6}(k_2' + k_3/T_m) \frac{R_d}{(\lambda + 1) g_m} e_0$$

whereby $T_m \cong T_0 [1 - \frac{1}{(\lambda+1)(\mu+1)}]$ can be computed from the temperature

Slant Delay

- The delay in the line of sight, or slant total delay, can be found by

$$STD = 10^{-6}\int_0^\infty N ds = 10^{-6}\int_{r_0}^\infty N\frac{ds}{dr}dr \qquad ; \quad \frac{ds}{dr} = \frac{1}{\cos z} \equiv \sec z$$

with z the zenith angle at each layer (changes because of curvature of the Earth). Approximations for ds/dr (ignoring the signal bending) or $\sec z$ (based on a Taylor expansion) are

$$\frac{ds}{dr} = r(r^2 - r_0^2\sin^2 z_0)^{-\frac{1}{2}} \quad ; \quad \sec z = \sec z_0 (1 - \tan^2 z_0 \frac{r - r_0}{r})$$

This is the approach used for the Saastamoinen (1972) model.

A more modern approach is to use **mapping functions**,

$$STD = m_h(z_0)\cdot ZHD + m_w(z_0)\cdot ZWD$$

with $m_h(z)$ and $m_w(z_0)$ the hydrostatic and wet mapping functions. Modern mapping functions are based on a continued fraction representation and some additional parameters.

Mapping functions

In the literature several mapping functions can be found:

- $m(z_0) = \sec z_0$ (Flat Earth society)
- $m(z_0) = \frac{1.001}{\sqrt{0.002001 + \cos^2 z_0}}$ (RTCA-MOPS, Collins, 1999)
- $m(z_0; a, b, c) = \frac{1 + a/(1 + b/(1 + c))}{\cos z_0 + \frac{a}{\cos z_0 + \frac{b}{\cos z_0 + c}}}$ (continued fractions)
- ...

Mapping functions using continued fractions

- Most modern mapping functions use continued fractions. The mapping function used by Ifadis (1992), Herring (1992) and Niell (1996) is

$$m(z_0; a, b, c) = \frac{1 + a/(1 + b/(1 + c))}{\cos z_0 + \cfrac{a}{\cos z_0 + \cfrac{b}{\cos z_0 + c}}}$$

with z_0 the zenith angle*). Clearly $m(0) = 1$. The coefficients have been computed from ray-tracing radiosonde and other data. For the hydrostatic and wet mapping functions different coefficients are used.

Ifadis (1992) gives the coefficients as function of P_0, T_0, and e_0, Herring (1992) gives them as function of latitude ϕ, height h_0 and T_0.

Niell (1996) uses values independent from surface meteorological data

$$a_h(\phi, doy) = a_{\mathrm{h;avg}}(\phi) + a_{\mathrm{h;}amp}(\phi)\cos[2\pi \frac{doy-28}{365.25}] \;\; ; \;\; a_w(\phi)$$

with $a_{h;avg}(\phi)$, $a_{h;amp}(\phi)$ and $a_w(\phi)$ tabulated for $\phi = 15^0, 30^0, 45^0, 60^0, 75^0$, with similar expressions for the other coefficients. The Niell mapping functions can be used for elevation angles down to 3^0.

*) In Marini (1972), Chao (1974) and Davis (1985) slightly different expressions were used.

Niell mapping function

coeff.	$\phi = 15°$	$\phi = 30°$	$\phi = 45°$	$\phi = 60°$	$\phi = 75°$
a_{avg}	1.2769934e-3	1.2683230e-3	1.2465397e-3	1.2196049e-3	1.2045996e-3
b_{avg}	2.9153695e-3	2.9152299e-3	2.9288445e-3	2.9022565e-3	2.9024912e-3
c_{avg}	62.610505e-3	62.837393e-3	63.721774e-3	63.824265e-3	64.258455e-3
a_{amp}	0.0	1.2709626e-5	2.6523662e-5	3.4000452e-5	4.1202191e-5
b_{amp}	0.0	2.1414979e-5	3.0160779e-5	7.2562722e-5	11.723375e-5
c_{amp}	0.0	9.0128400e-5	4.3497037e-5	84.795348e-5	170.37206e-5
a_{wet}	5.8021897e-4	5.6794847e-4	5.8118019e-4	5.9727542e-4	6.1641693e-4
b_{wet}	1.4275268e-3	1.5138625e-3	1.4572752e-3	1.5007428e-3	1.7599082e-3
c_{wet}	4.3472961e-2	4.6729510e-2	4.3908931e-2	4.4626982e-2	5.4736038e-2

$$m_h(z_0;\phi,doy,h_0) = m(z_0;a_h,b_h,c_h) + [\sec z_0 - m(z_0; 2.53\ 10^{-5}, 5.49\ \ 10^{-3}, 1.14\ 10^{-3})]\ h_0\ [km]$$

$$a_h(\phi,doy) = a_{\mathrm{h;avg}}(\phi) + a_{\mathrm{h};amp}(\phi)\cos[2\pi\frac{doy-28}{365.25}] \quad ; \quad a_w(\phi)$$

$$m(z_0;a,b,c) = \frac{1+a/(1+b/(1+c))}{\cos z_0 + \cfrac{a}{\cos z_0 + \cfrac{b}{\cos z_0 + c}}}$$

Troposphere Slant Delay - Conclusion

- The Slant Tropsphere Delay (STD) can be written as

$$STD = m_h(z_0) \cdot ZHD + m_w(z_0) \cdot ZWD$$

with $m_h(z)$ and $m_w(z_0)$ the 'known' hydrostatic and wet mapping function.

The Zenith Hydrostatic Delay (ZHD) is a function of the **pressure** P_0 $[hPa]$ at **height** h_0 and **latitude** ϕ, with

$$ZHD = 10^{-6} \int_{h_0}^{\infty} N_h(h) dh \quad \cong 0.0022768 \frac{P_0}{f(\phi, h_0)}$$

with $f(\phi, h_0) = 1 - 2.66\ 10^{-3} cos2\phi - 2.8\ 10^{-7}\ h_0 \ \approx 1$.

The Zenith Wet Delay (ZWD) is a function of Integrated Water Vapour **IWV** [kg/m^2] and **mean temperature** T_m

$$ZWD = 10^{-6}(k_2' + k_3/T_m)R_v\ IWV \cong Q(T_s)\ IWV$$

Use of NWP data for correcting GPS data

- New generation of mapping functions based on ECMWF data (IMF, GMF, VMF1)

- Direct use of NWP products (to reduce systematic errors)
  - Mapping function replacement (e.g. Vienna Mapping Function - VMF1
  - A-priori zenith hydrostatic delay
  - ➔ Zenith wet delay is still to be estimated

- Direct use of NWP products (for Network RTK)
  - ZTD, ZWD and Mapping function
  - Direct computation of slant delay
  - ➔ No troposphere parameters have to be estimated

Vienna Mapping Functions (VMF1)

[Boehm et. al, 2006, 2007, 2008]

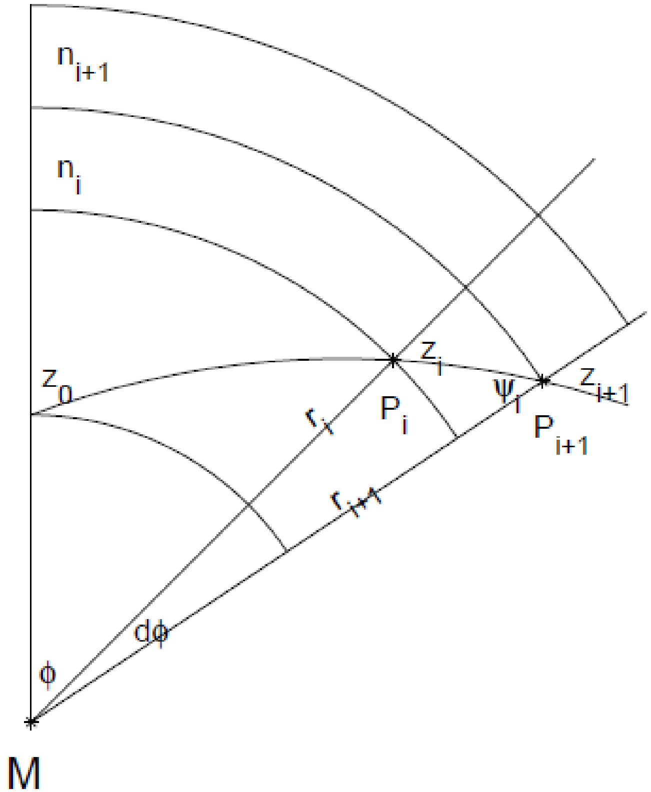
- Mapping functions in continued fraction (with *a*, *b*, and *c* coefficients) and accompanying zenith delays derived from ECMWF reanalysis
- Hydrostatic *b* and *c* coefficients derived from a lsq fit to ray-traced observations at 9 elevation angles; *b=0.0029* and *c* is a 3-parameter function (fixed values) of latitude and day of year.
- The wet (non-hydrostatic) *b* and *c* coefficients are the same as NMF [Niell, 1996] at 45^o latitude, and equal to 0.00146 and 0.04391.
- The hydrostatic and wet *a* coefficients are determined from direct ray tracing through the ECMWF analysis on a 6 hour basis at initial elevation angle 3.3^0 (the outgoing angle will be approx. 3^0), and then inverting the continued fraction equation using the previously determined *b* and *c* coefficients.
- The *a* coefficients are provided on a 2.0^o X 2.5^o lat/lon grid at 0, 6, 12, and 18 UT for zero height (user need to correct for station height).
- The ZHD and ZWD are also provided in gridded format at mean grid height given by the orography file (user must correct to station height).

Ray-tracing in the VMF1

For a spherical earth with spherical layered atmosphere Snell's law we have

$$n\, r \sin z = constant$$

The hydrostatic and wet *a* coefficients are determined from direct ray tracing through the ECMWF analysis on a 6 hour basis at initial elevation angle 3.3^0 (the outgoing angle will be approx. 3^0). Ray-tracing through the model layers gives the slant delay, as well as the mapping function value (slant / zenith delay). Inverting the continued fraction equation, using the empirically determined *b* and *c* coefficients, gives the *a* coefficient.



$$\frac{r_i}{sin\psi_i} = \frac{r_{i+1}}{\sin(\pi - z_i)} = \frac{r_i}{sin z_i}$$

Figure courtesy of Frank Kleijer, 2004.

Vienna Mapping Functions (VMF1) – cont’

- Originally the VMF1 was produced on a site specific basis -> but use of gridded data is now recommended
- Gridded VMF1 coefficients are available with for each parameter four files per day, i.e. at 0, 6, 12, and 18 UT, with a latency of 2.5 days
- Grids are also determined from forecast data of the ECMWF, but in this case a password is needed.
- GMF (Global Mapping Function) is a 'backup' mapping function for VMF1 similar to Niell NMF mapping function (input parameters are day of year, latitude, longitude and height).
- In 2012 UNB started to provide UNB-VMF1, which is a independent solution based on the NOAA-NCEP model and alternative ray-tracing algorithm [Urquhart et.al., 2011]. The agreement with VMF1 is very good.
- Using VMF1 or UNB-VMF1 can affect the estimated station heights by +- 2 mm (10 mm in the worst case) when compared to NMF; estimated ZWD are affected as well (factor 3 larger). [Kouba, 2008]

Vienna Mapping Functions (VMF1) – extra’s

Some extra data can be provided from the ECMWF model:

- An empirical model to determine pressure and temperature from the station coordinates and the day of the year.
- The height of the 200 hPa pressure level which is the input parameter for the hydrostatic IMF is provided on a grid.
- The mean temperature of the atmosphere (in Kelvin) which can be used to convert the wet zenith delays into precipitable water is provided on a grid
- Hydrostatic and wet linear horizontal gradients are provided for the same stations where VMF1 site is available.