Tomography (GNSS)

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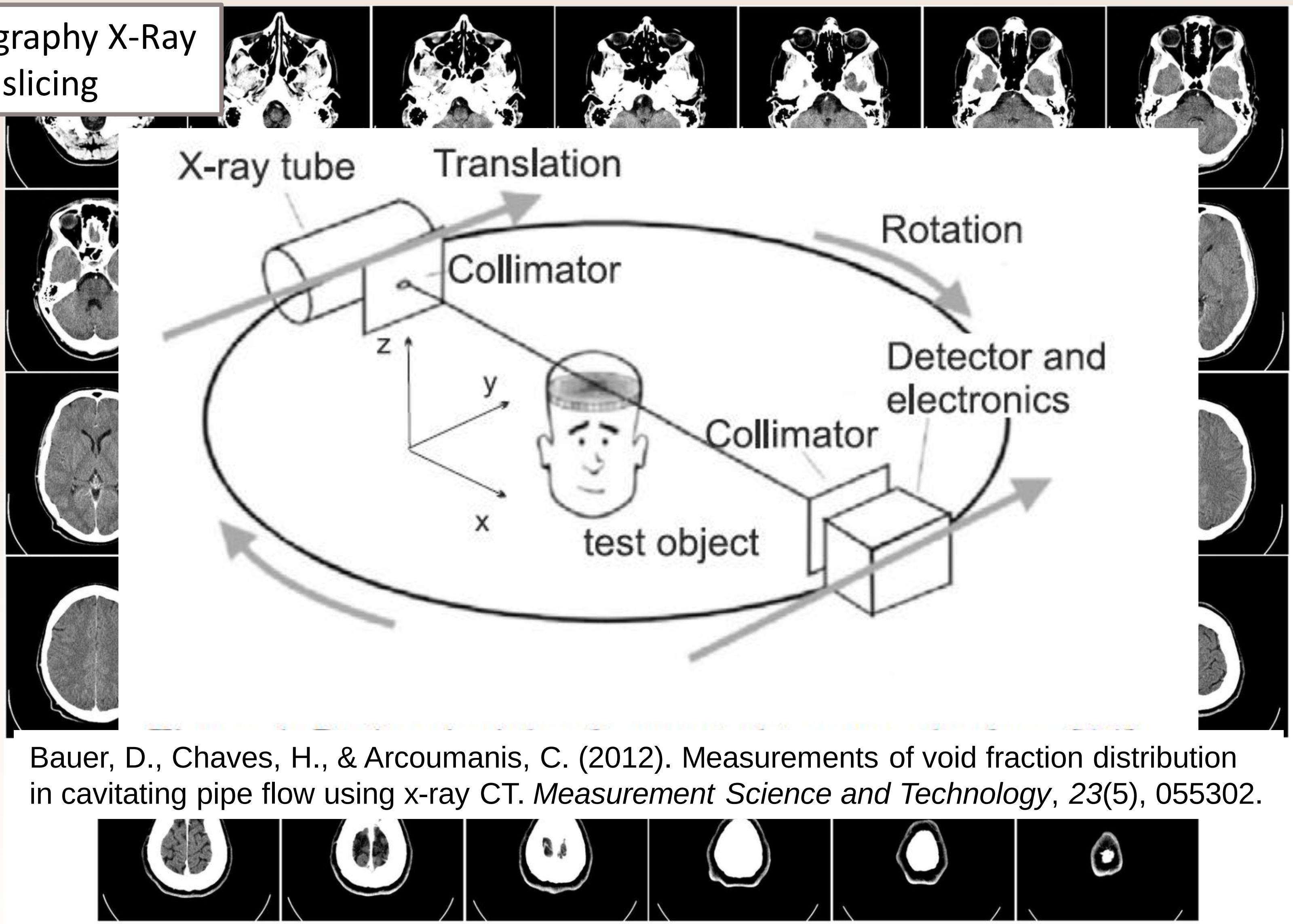
Tomography: why it is so cool?

Tomography X-Ray slicing



Bauer, D., Chaves, H., & Arcoumanis, C. (2012). Measurements of void fraction distribution in cavitating pipe flow using x-ray CT. *Measurement Science and Technology*, *23*(5), 055302.

Tomography: why it is so cool?

X-Ray Vision for Robots: Seeing Through Walls with Only WiFi

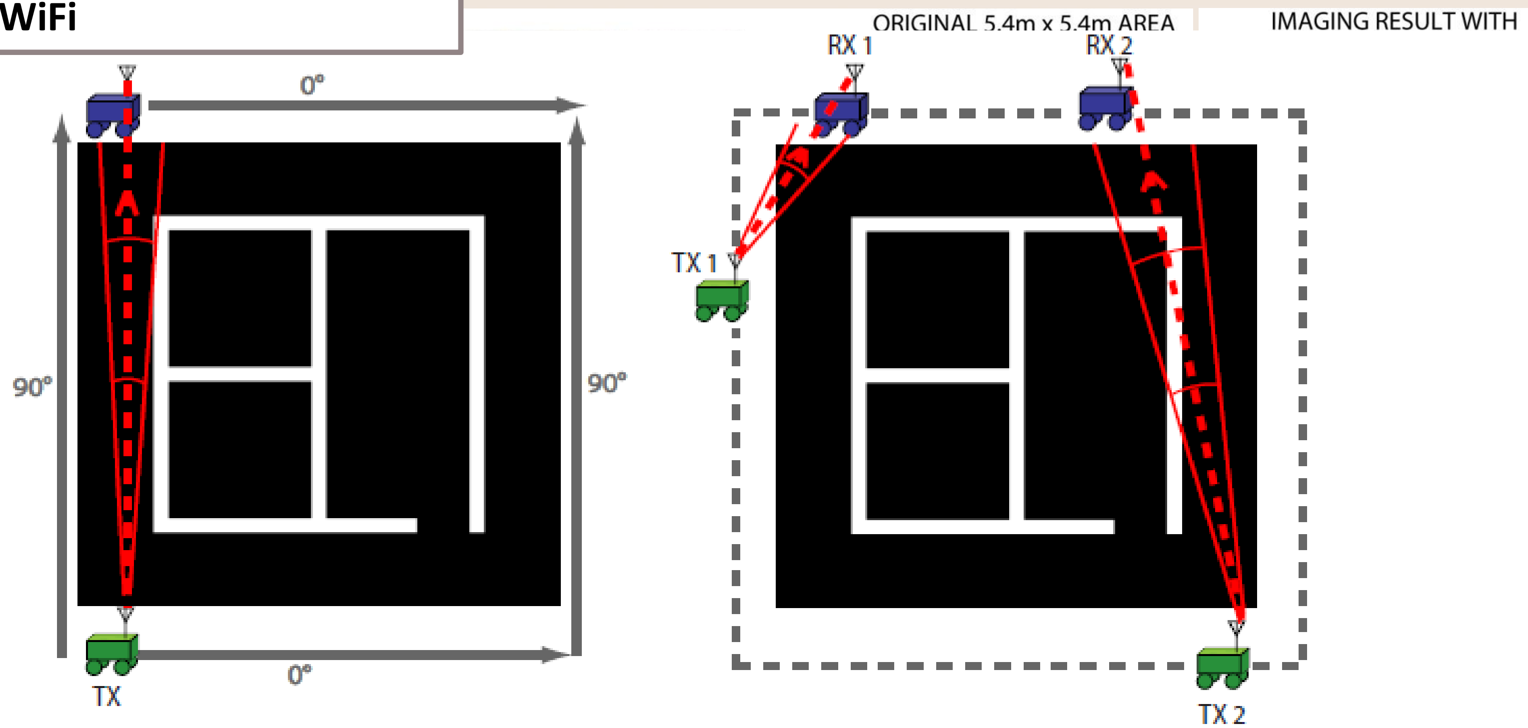


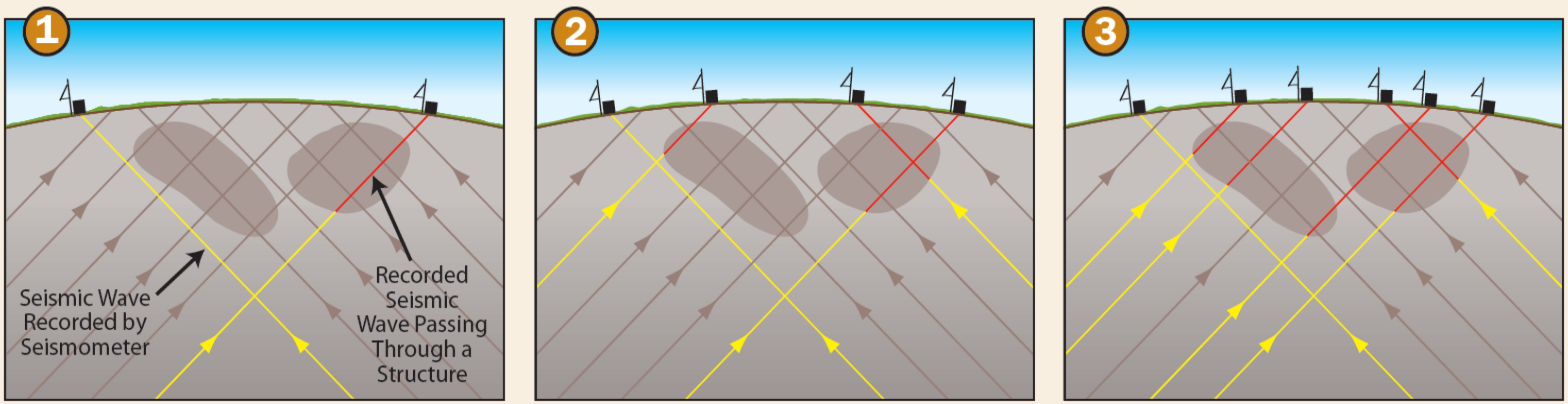
Fig. 3. An illustration of wireless-based obstacle mapping with (left) coordinated wireless measurements and (right) random wireless measurements.

Gonzalez-Ruiz, A., Ghaffarkhah, A., & Mostofi, Y. (2014). An Integrated Framework for Obstacle Mapping with See-Through Capabilities using Laser and Wireless Channel Measurements. *IEEE SENSORS JOURNAL*, *14*(1), 25-38.

http://www.ece.ucsb.edu/~ymostofi/SeeThroughImaging.html#Publications

Tomography: why it is so cool?

> IMAGING THE EARTH WITH SEISMIC WAVES



(1) Two seismometers on the surface record incoming seismic waves. Only one recorded wave passes through part of one of the structures with different properties than the surrounding material. Scientists examining the recorded seismograms would infer that only one structure is present instead of two.

(2) As more seismometers are added, scientists can detect two structures from the recorded seismograms, but they can't determine the size and shape of each.

(3) Six seismometers catch enough recorded waves for scientists to start determining the borders of the structures. With dozens of seismometers, scientists can produce an image of a slice of the Earth.

http://www.iris.edu/hq/files/programs/education_and_outreach/lessons_and_resources/docs/es_tomography.pdf

Tomography: why it is so cool?

- You don’t need to cut open somebody’s head to research it (X-ray tomography)
- You can develope new applications of old signals (e.g. WiFi – see-through)
- You can research the earth crust without diging a hole (Seismic tomography)
- Can you picture atmosphere with GNSS and tomography?

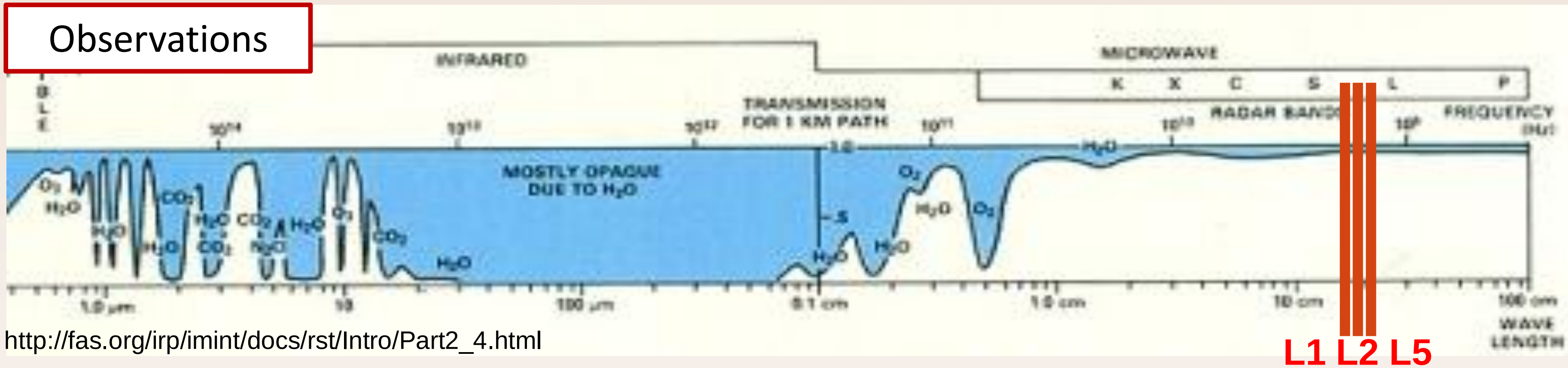
Lecture content

- Tomography principles
- GNSS troposphere tomography
  - Building blocks
  - Equation system (link)
  - Observations
  - Constraints
  - Implementations
- Applications in meteorology
- Future development

Preconditions for successful tomography

- The waves that penetrates the substance of interest are **partially** sensitive to the molecules of that substance

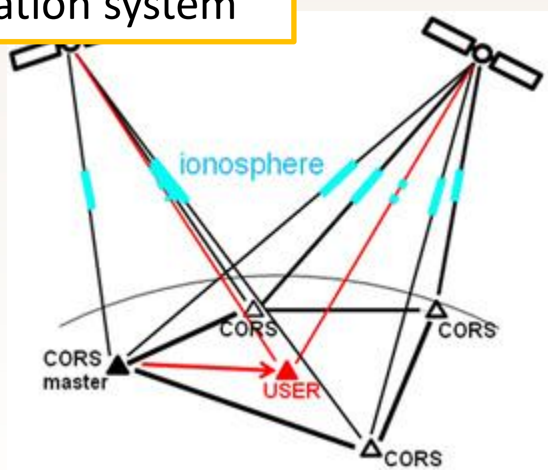
Observations



http://fas.org/irp/imint/docs/rst/Intro/Part2_4.html

- Waves receivers that are located on the opposite side of the investigated structure

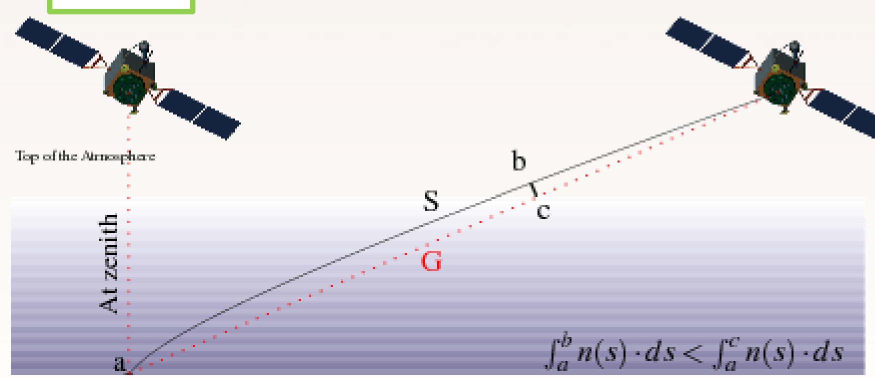
Observation system



http://gnss.curtin.edu.au/research/cors_rtk.cfm

- Waves propagation effects known as an analytical or empirical formulas

Link



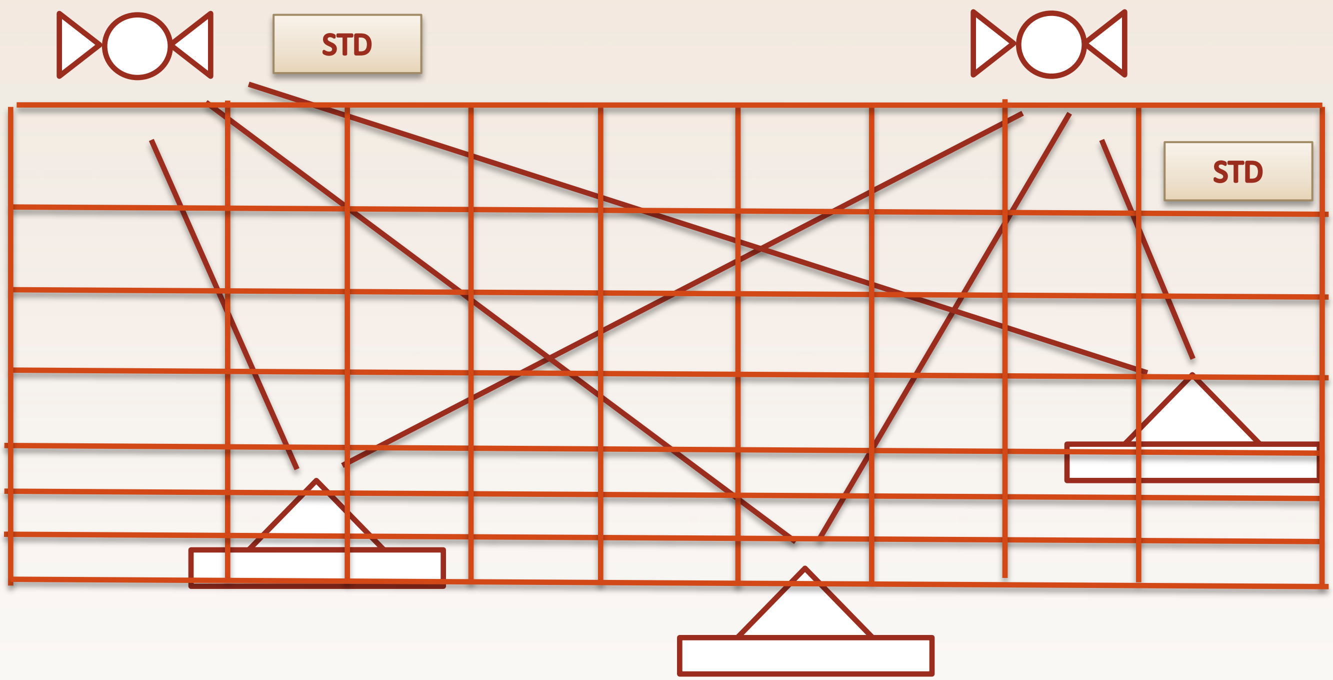
http://gnss.be/troposphere_tutorial.php

GNSS Tomography: building blocks

Observations

Observation system

Link



Tomography: link between troposphere and delay

Link

TOTAL NEUTRAL ATMOSPHERE DELAY

$$N_0 = k_1 \frac{p-e}{T} \cdot Z_d^{-1} + \left(k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right) \cdot Z_v^{-1} = dry + WV$$

$$N_0 = k_1 \frac{P}{T_v} + k_2' \frac{e}{T} + k_3 \frac{e}{T^2} = hydrostatic + non-hydrostatic(wet)$$

$$STD = 10^{-6} \int N_o ds = 10^{-6} \sum N_o \Delta s = 10^{-6} \sum N_o d$$

WATER VAPOUR REFRACTIVITY DELAY/ WET REFRACTIVITY

$$N_w = k_2' \frac{e}{T} + k_3 \frac{e}{T^2}$$

$$SWD = 10^{-6} \int N_w ds = 10^{-6} \sum N_w \Delta s$$

$$N_v = \left(k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right) \cdot Z_v^{-1}$$

$$SWVD = 10^{-6} \int N_v ds = 10^{-6} \sum N_v \Delta s$$

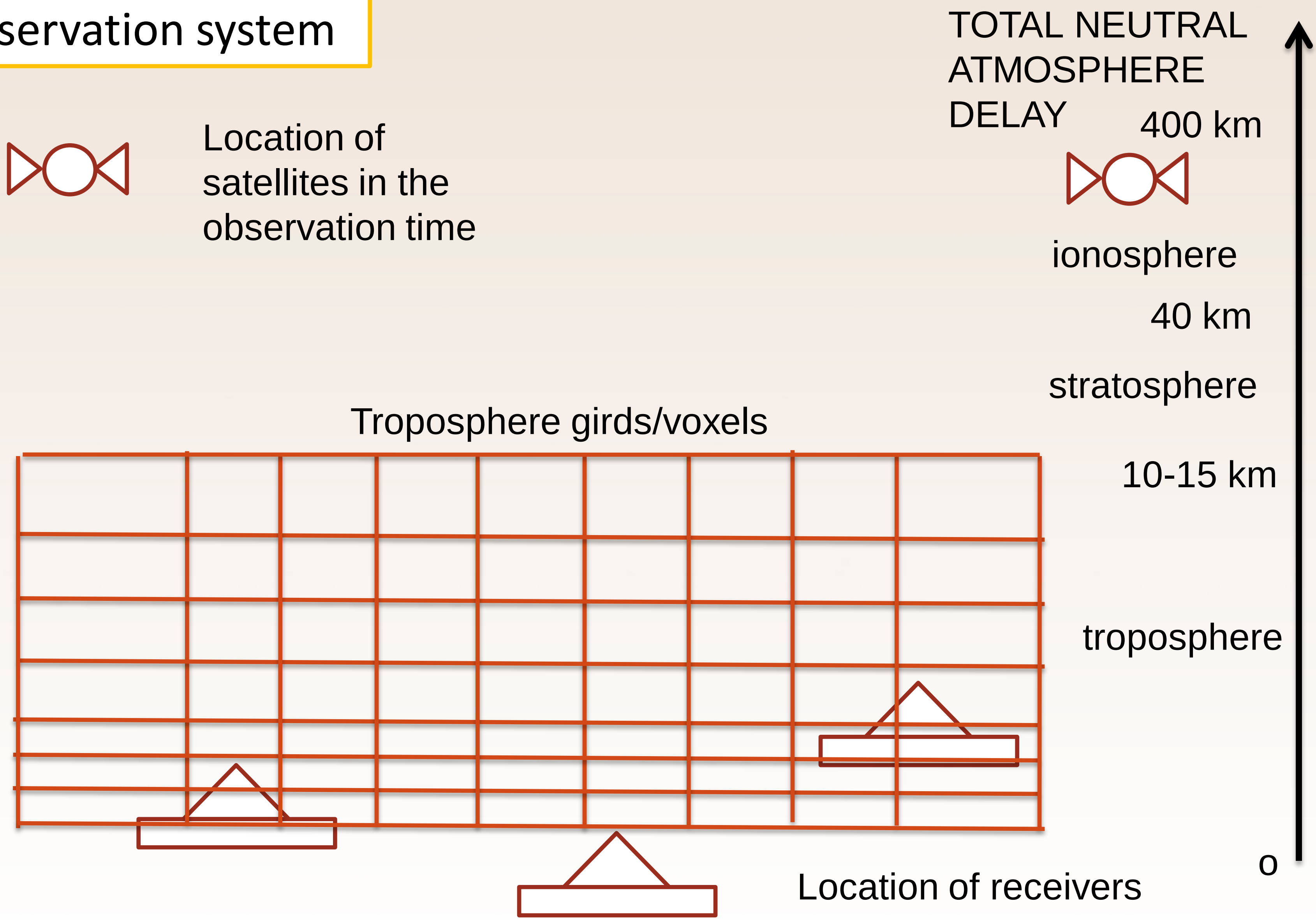
PWAT

$$PW = \frac{\mathrm{p}}{\mathrm{R} \cdot (1 + 0.61 \cdot \mathrm{q})} \mathrm{T} \cdot \mathrm{q} \cdot dz$$

$$SPWD = \int PW ds = \sum PW \Delta s$$

Tomography: structure

Observation system



Tomography: observations (1)

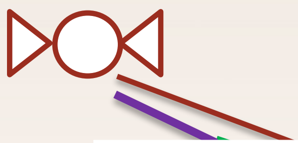
Observations

$$STD = \delta\rho_{apr,R}(z_R^S) + \delta^h\rho_R(t)\cdot mf_N(z_R^S) + \delta^n\rho_R(t)\cdot\frac{\partial mf_N}{\partial z}\cos A_R^S + \delta^e\rho_R(t)\cdot\frac{\partial mf_N}{\partial z}\sin A_R^S + ZDres$$

ZTD

ZTD Gradients

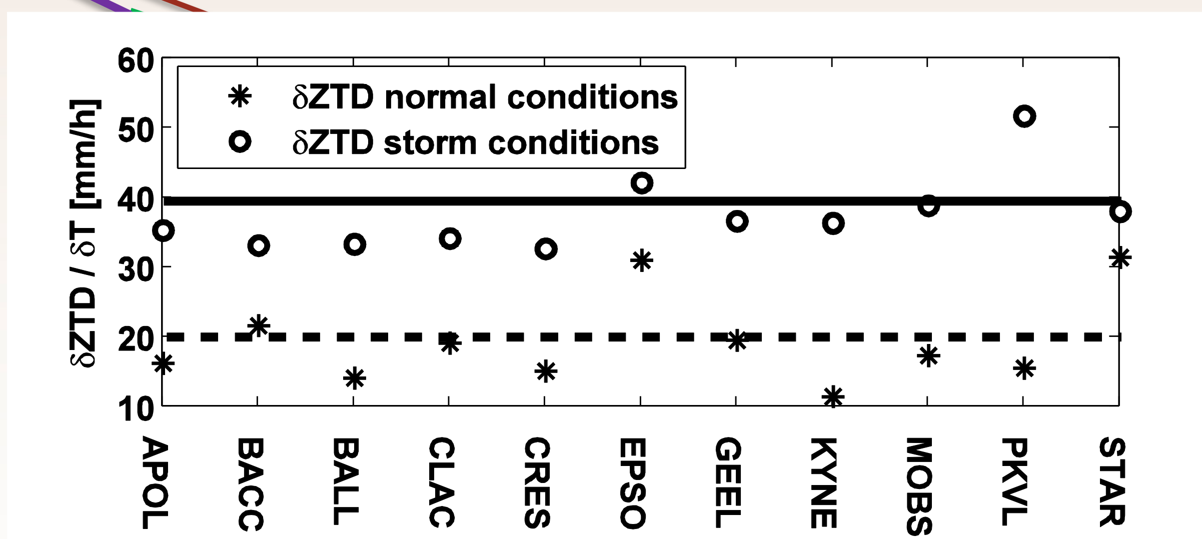
ZD Residuals



Stochastic modelling

Functional modelling

$$\frac{d\delta^h\rho_R}{dt} = \frac{\delta^h\rho_R}{t} + w(t)$$



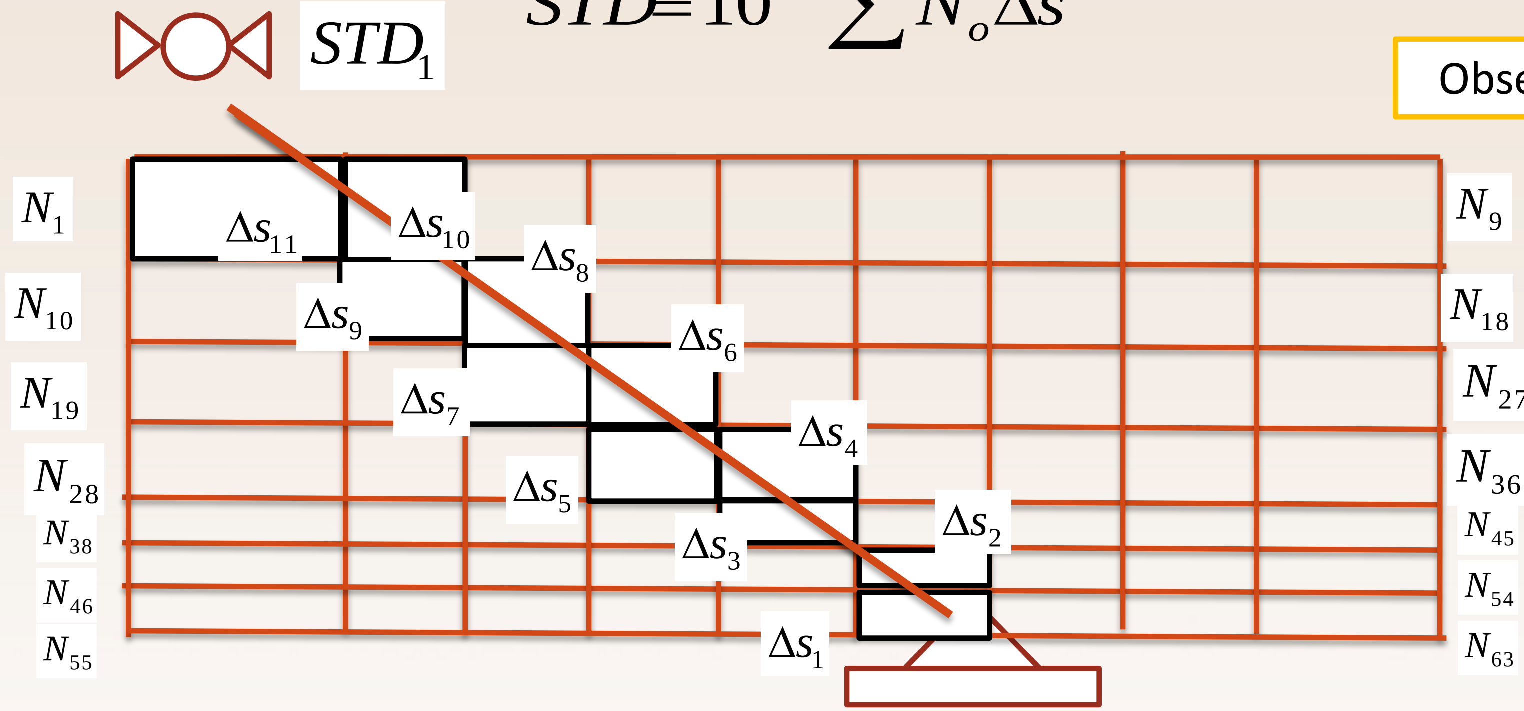
Tomography: single observation (2)

Observations

Link

Observation system

$$STD = 10^{-6} \sum N_o \Delta s$$



$$STD = 10^{-6} \cdot (N_{60} \cdot \Delta s_1 + N_{51} \cdot \Delta s_2 + N_{41} \cdot \Delta s_3 + N_{32} \cdot \Delta s_4 + N_{31} \cdot \Delta s_5 + N_{22} \cdot \Delta s_6 + N_{21} \cdot \Delta s_7 + \ldots.$$

$$\ldots. + N_{12} \cdot \Delta s_8 + N_{11} \cdot \Delta s_9 + N_2 \cdot \Delta s_{10} + N_1 \cdot \Delta s_{11})$$

Tomography: multiple observations (3)

Observations

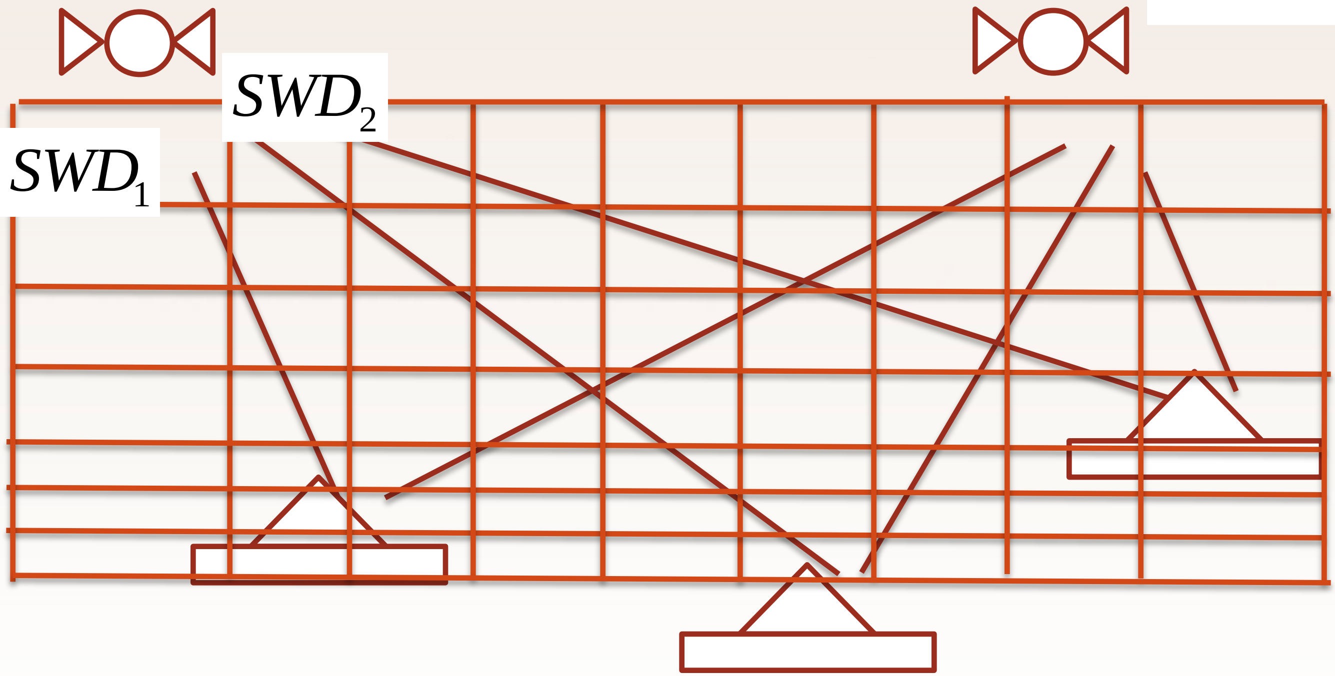
$$SWD = \begin{bmatrix} SWD_1 \\ SWD_2 \\ \vdots \\ SWD_n \end{bmatrix}$$

Tomography model structure

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & 0 & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & 0 & d_{n4} & \cdots & d_{nm} \end{bmatrix}$$

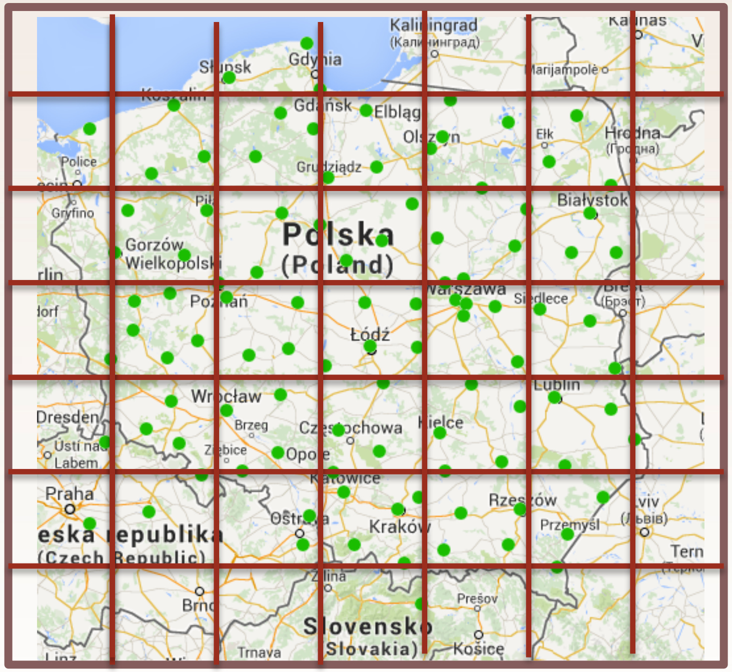
Unknowns

$$N_w = \begin{bmatrix} N_{w1} \\ N_{w2} \\ \vdots \\ N_{wm} \end{bmatrix}$$



Tomography: problem ill-posedness

EXAMPLE POLAND
120 ZTDs every hour ~600 SWDs ,
Number of unknowns (10x12x10) =
1200 voxels
A matrix is sparse
SWDs are correlated



$$SWD = A \cdot N_v$$

$$[600x1] = [600x1200] \cdot [1200x600]$$

Solve the system:

$$N_v = (A^T A)^{-1} A^T SWD$$

Add weights (P) and constraints (B)

$$N_v = (A^T \cdot P \cdot A + B^T \cdot B)^{-1} A^T \cdot P \cdot SWD$$

Use pseudo inverse not unique but optimal (SVD)

$$N_v = (A^T \cdot P \cdot A + B^T \cdot B)^{+} A^T \cdot P \cdot SWD$$

Select best singular values with (TSVD)

$$N_v = (A^T \cdot P \cdot A + B^T \cdot B)^{+} A^T \cdot P \cdot SWD$$

Tomography: constraints(1)

Observations

$$SWD = \begin{bmatrix} SWD_1 \\ SWD_2 \\ \vdots \\ SWD_n \end{bmatrix}$$

$$w_{ij} = dist(voxel_i, voxel_j)$$

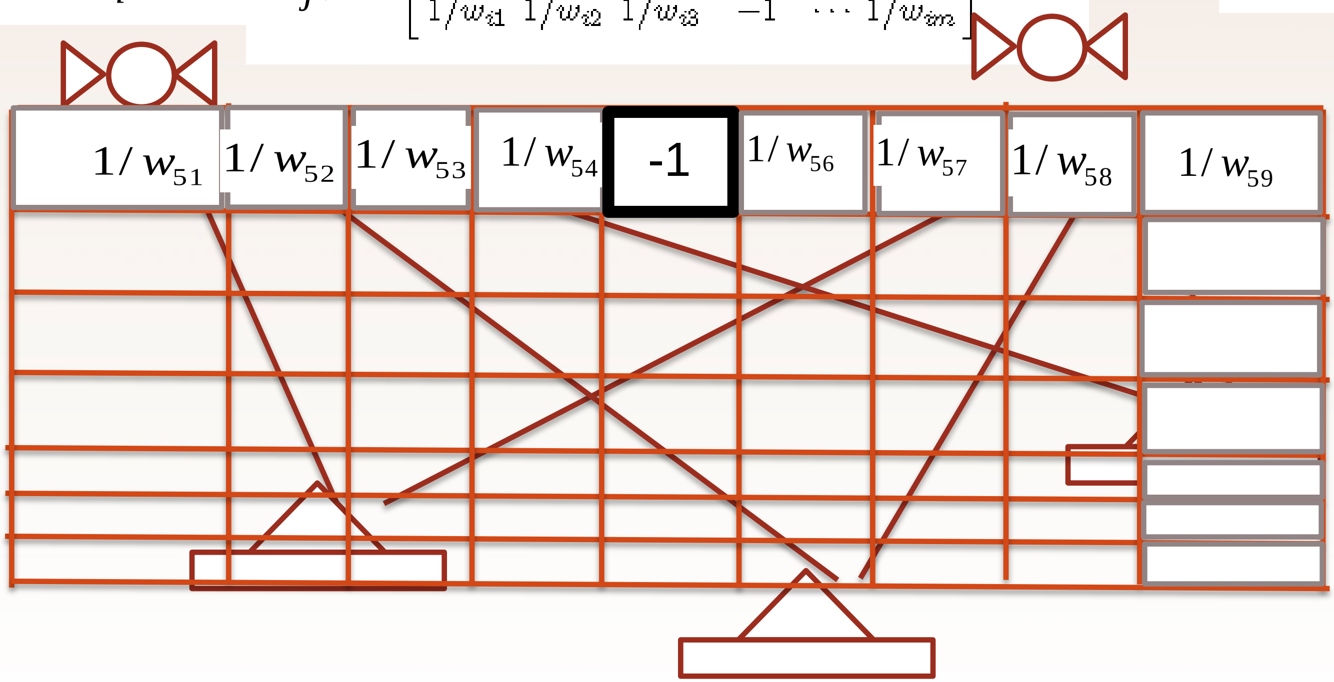
Tomography model structure

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & 0 & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & 0 & d_{n4} & \cdots & d_{nm} \\ -1 & 1/w_{12} & 1/w_{13} & 1/w_{14} & \cdots & 1/w_{1m} \\ 1/w_{21} & -1 & 1/w_{23} & 1/w_{24} & \cdots & 1/w_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1/w_{i1} & 1/w_{i2} & 1/w_{i3} & -1 & \cdots & 1/w_{im} \end{bmatrix},$$

Unknowns

$$N_w = \begin{bmatrix} N_{w1} \\ N_{w2} \\ \vdots \\ N_{wm} \end{bmatrix}$$

Horizontal constraints



Vertical constraints

Tomography: pseudo observations and outer (1)

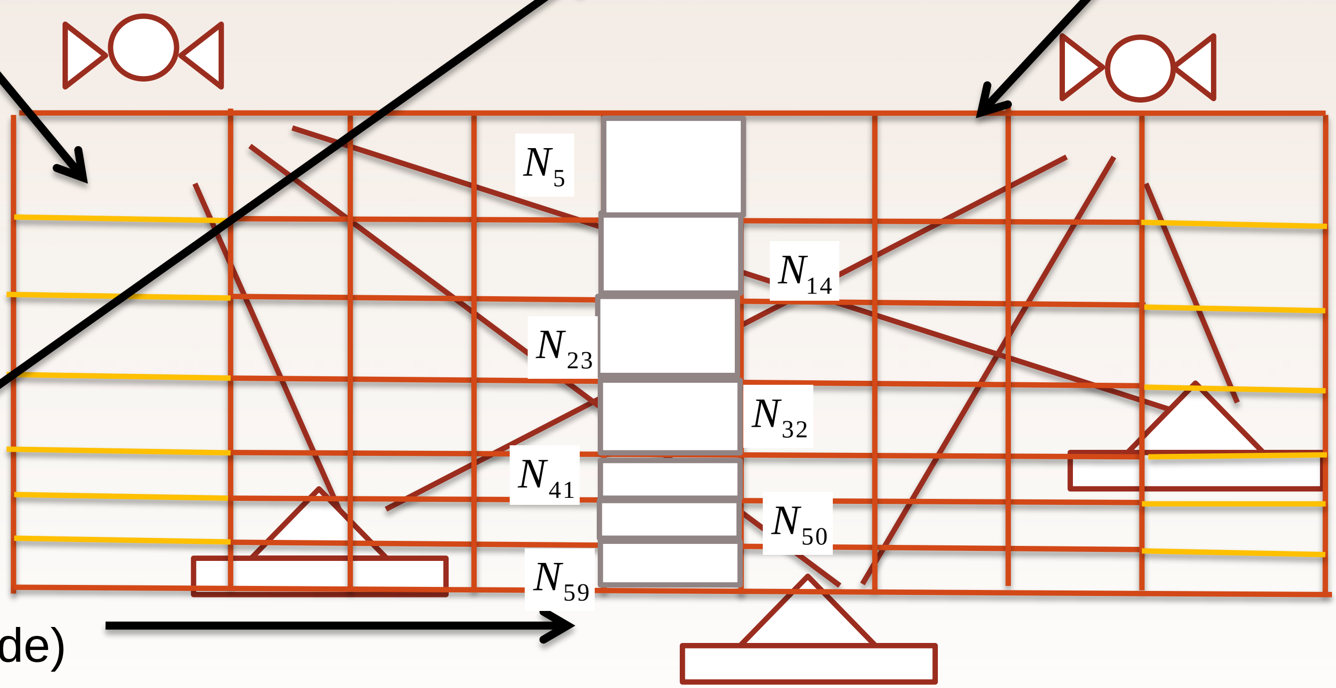
Tomography model structure

Observations | Unknowns

$$\begin{bmatrix} \mathrm{SWD} \\ \mathrm{N}_{\mathrm{vapriori}} \end{bmatrix} = \begin{bmatrix} A_{inner} & A_{outer} \\ A_{apriori} & 0 \end{bmatrix} \begin{bmatrix} N_v \\ No_v \end{bmatrix} + v$$

Outer model domain

Inner model domain



Observations
(e.g. radiosonde)

Tomography implementations: WLS

$$N_w = (A^T \cdot P \cdot A)^+ \cdot A^T \cdot P \cdot SWD^T$$

(Flores et al., 2000)

Close to singular, ill-posed system

Pseudo-inverse

$$pinv\left(A^T \cdot P \cdot A\right)^+ = V \cdot S^+ \cdot U^T$$

$$C_x = \begin{bmatrix} m_{ZTD}^2 & 0 & 0 & 0 & 0 \\ 0 & m_p^2 & 0 & 0 & 0 \\ 0 & 0 & m_h^2 & 0 & 0 \\ 0 & 0 & 0 & m_\epsilon^2 & 0 \\ 0 & 0 & 0 & 0 & m_{map}^2 \end{bmatrix}.$$

$$C_{SWD} = J_{SWD} \cdot C_x \cdot J_{SWD}^T$$

Uncertainty of ZTD 0.004 - 0.007 m converts to Uncertainty of SWD 0.008 – 0.050 m (in optimal conditions)

Add wieghts for constraints and pseudo observations

Tomography implementations: KF/RKF

$$\begin{bmatrix} SWD \\ N_{wapriorir} \\ 0 \end{bmatrix} = \begin{bmatrix} A \\ A_{apriori} \\ W \end{bmatrix} \cdot N_w$$

(Rohm et al., 2014)

State predicition

I

State first guess

$$\hat{N}w_k(-) = \Phi_k \cdot \hat{N}w_{k-1}(+)$$

Corrected state estimate

$$\hat{N}w_k(+) = \hat{N}w_k(-) + K_k \cdot \left(SWD_k - A_k \cdot \hat{N}w_k(-)\right)$$

Predicted

$$Q_k(-) = \Phi_k \cdot Q_{k-1}(+) \cdot \Phi_k^t + Nproc_{k-1}$$

Process noise

Corrected state covariance

Kalman Gain matrix (Robust KF iterative process)

$$K_k = Q_k($$

$$Q_k(+) = Q_k(-) - K_k \cdot A_k \cdot Q_k(-)$$

Design matrix

Inversion process

Observation noise

Robust KF - modification

Tomography implementations: (M)ART

Iterative algorithm with two loops (Bender et al., 2009)

outer k = 1 : number of iterations

Inner i = 1 : number of rows in A (observations)

SWD

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \lambda \frac{m_i - \langle \boldsymbol{A}^i, \boldsymbol{x}^k \rangle}{\langle \boldsymbol{A}^i, \boldsymbol{A}^i \rangle} \boldsymbol{A}^i$$

N_w

Good quality of initial field (x) required as the updates only are estimated

No inversion

λ important to choose apropriate value

outer k = 1 : number of iterations

Inner i = 1 : number of rows in A (observations)

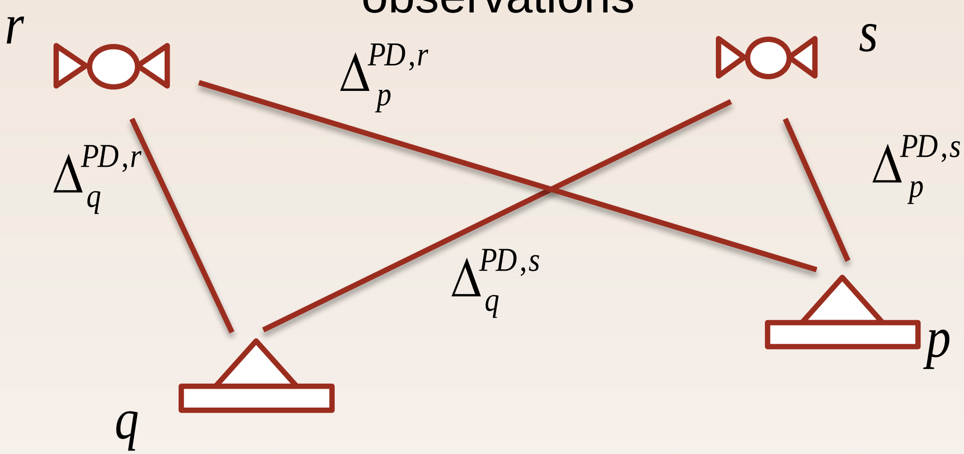
Inner j = 1 : number of columns in A (unknowns)

$$x_j^{k+1} = x_j^k \cdot \left(\frac{m_i}{\langle \boldsymbol{A}^i, \boldsymbol{x}^k \rangle} \right)^{\frac{\lambda A_j^i}{\sqrt{\langle A^i, A^i \rangle}}} \quad \text{with} \quad j = 1, \ldots, N.$$

Tomography implementations: DD obs + spline

observations

(Perler et al., 2011)



$$\Delta_{pq}^{2,PD,rs} = \left(\Delta_q^{PD,r} - \Delta_p^{PD,r}\right) - \left(\Delta_q^{PD,s} - \Delta_p^{PD,s}\right)$$

$$\Delta_{\mathrm{wet},pq}^{2,PD,rs} = \overline{\Delta_{\mathrm{wet},pq}^{2,PD,rs}} + \Delta_{pq}^{2,RES,rs}$$

parametrisation

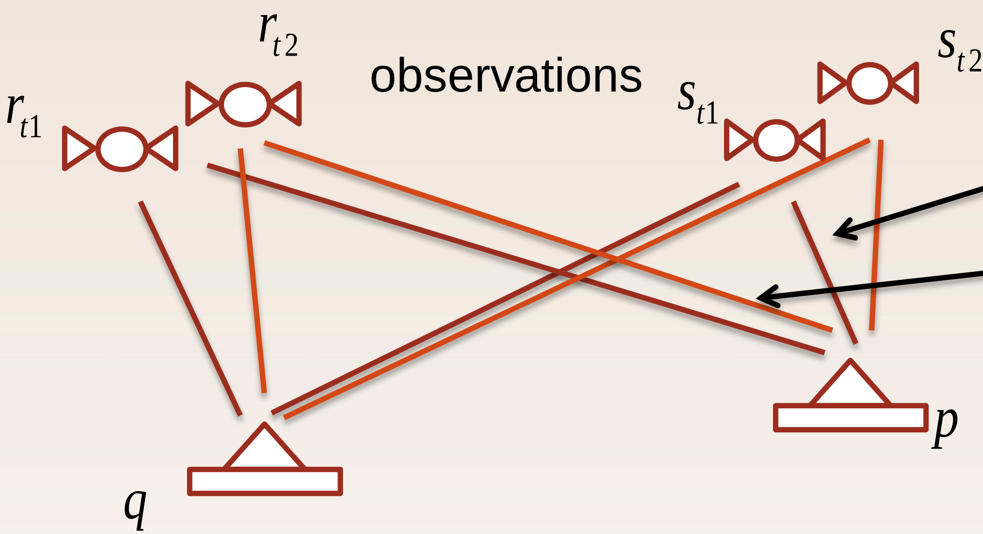
$$\mathrm{N}(\lambda,\phi,h) = \boldsymbol{\omega}^T \begin{bmatrix} \mathrm{N}_k(\lambda,\phi) \\ \mathrm{N}_{k+1}(\lambda,\phi) \\ \mathrm{N}_k''(\lambda,\phi) \\ \mathrm{N}_{k+1}''(\lambda,\phi) \end{bmatrix}$$

with

$$\boldsymbol{\omega} = \begin{bmatrix} 1 - \frac{h-h_k}{dh_k} \\ \frac{h-h_k}{dh_k} \\ \frac{(h-h_k)^2}{2} - \frac{dh_k(h-h_k)}{3} - \frac{(h-h_k)^3}{6dh_k} \\ \frac{(h-h_k)^3}{6dh_k} - \frac{dh_k(h-h_k)}{6} \end{bmatrix},$$

Tomography implementations: stacking

(e.g. Brenot et al., 2014; Rohm, 2013)



$$\begin{bmatrix} \delta SWD_1 \\ \delta SWD_2 \\ \cdots \\ \delta SWD_p \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ \cdots \\ A_p \end{bmatrix} \cdot \delta N_w .$$

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & 0 & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & 0 & d_{n4} & \cdots & d_{nm} \end{bmatrix}$$

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & 0 & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & 0 & d_{n4} & \cdots & d_{nm} \end{bmatrix}$$

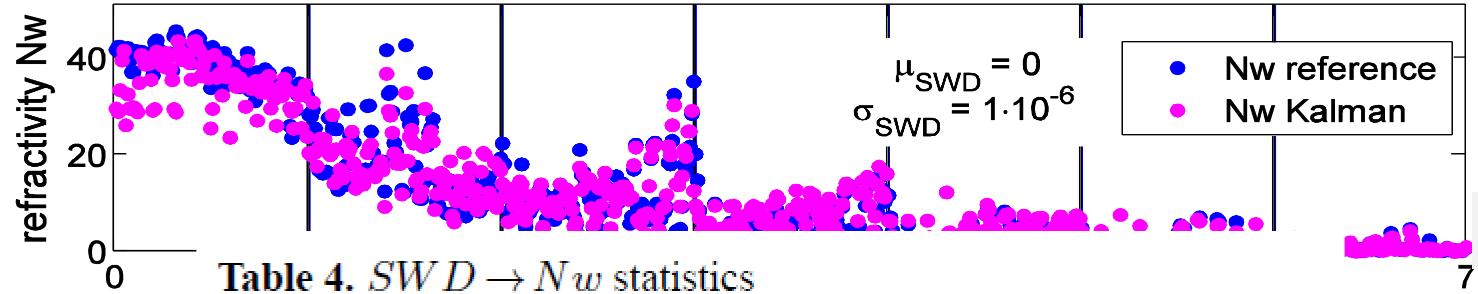
$$N_w = \begin{bmatrix} N_{w1} \\ N_{w2} \\ \vdots \\ N_{wm} \end{bmatrix}$$

There are „n” observation matrices
There are „n” design matrices
There is **only one Nw** (or expressed as a some function)

How long you can accumulate the STD data as a result of one „atmosphere state”?

Tomography retrieval quality

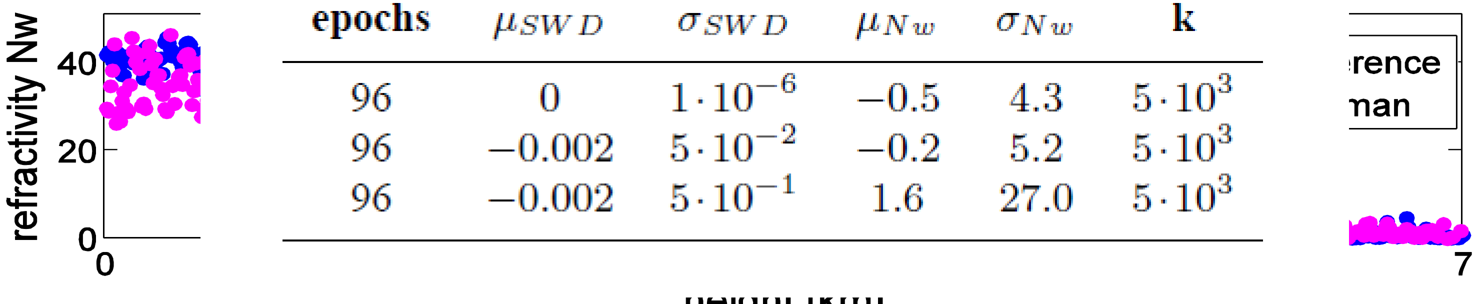
Robust Kalman Filter



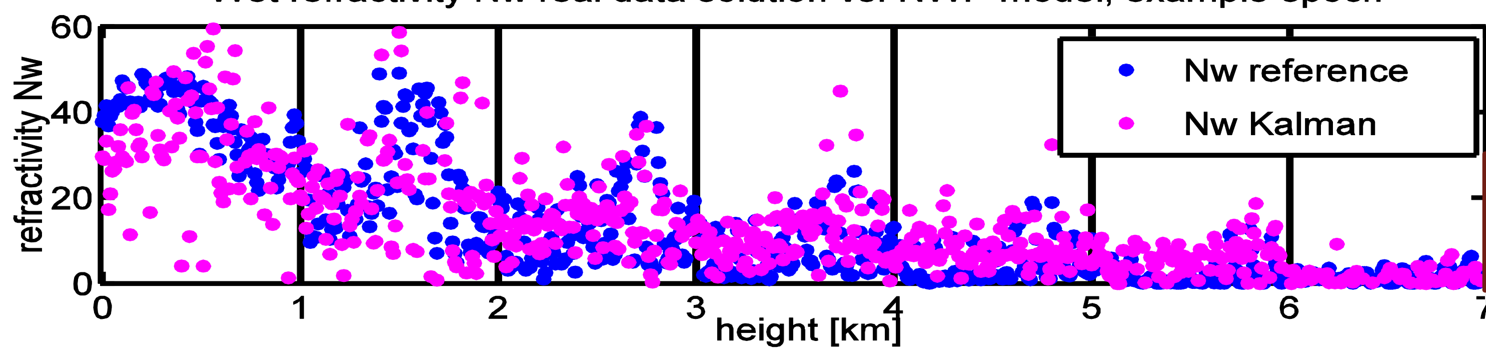
zero noise

Table 4. $SWD \rightarrow Nw$ statistics

epochs	μ_{SWD}	σ_{SWD}	μ_{Nw}	σ_{Nw}	k
96	0	$1\cdot10^{-6}$	-0.5	4.3	$5\cdot10^{3}$
96	-0.002	$5\cdot10^{-2}$	-0.2	5.2	$5\cdot10^{3}$
96	-0.002	$5\cdot10^{-1}$	1.6	27.0	$5\cdot10^{3}$

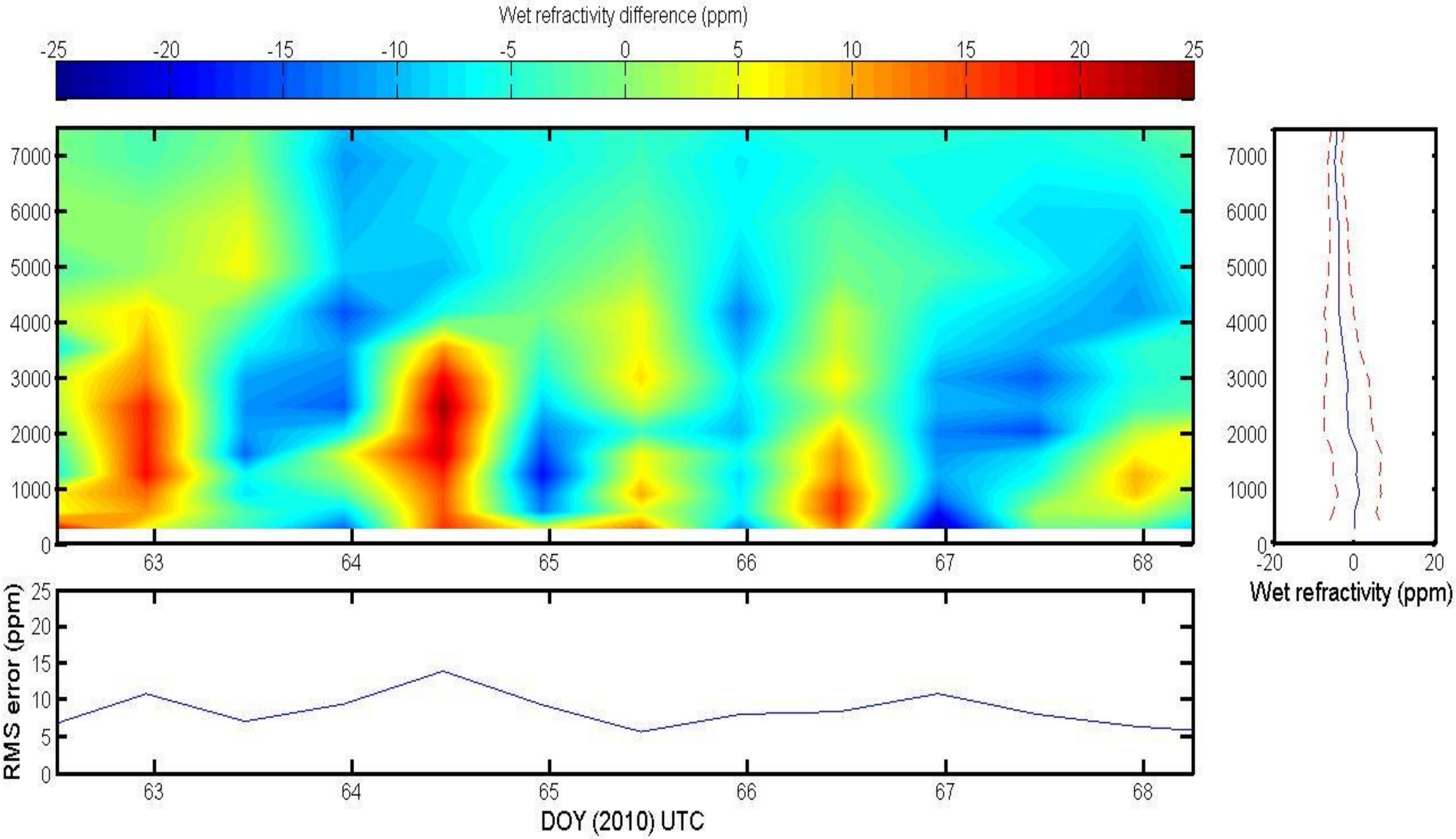


low noise

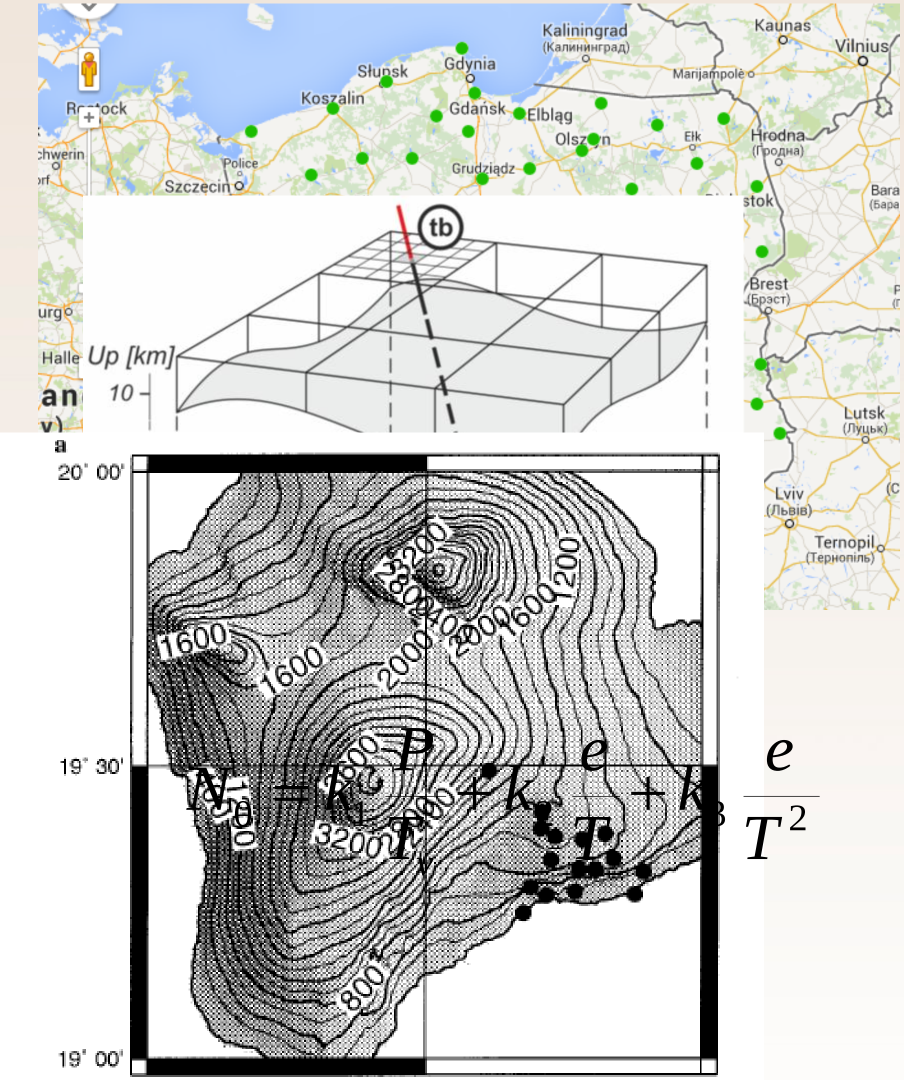


noise reduction

Tomography retrieval quality



Tomography: practical considerations



$$N_0 = k_1 \frac{P}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

Flores, A., Ruffini, G., & Rius, A. (2000, February). 4D tropospheric tomography using GPS slant wet delays. In *Annales Geophysicae* (Vol. 18, No. 2, pp. 223-234). Springer-Verlag.

ZTD to SWD conversion supported with pressure information – NWP is a reasonable choice of pressure data

The size of the voxels should not be smaller than half the distance between stations

Heavy undulated areas are better for tomography.

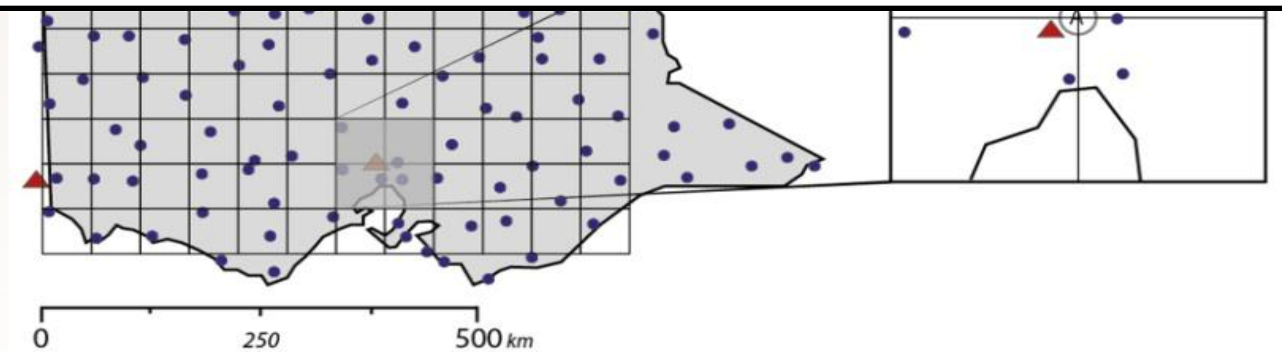
A matrix condition number is a good approximation of the tomography geometry quality

$$cond(A) = \frac{s_k}{s_1}$$

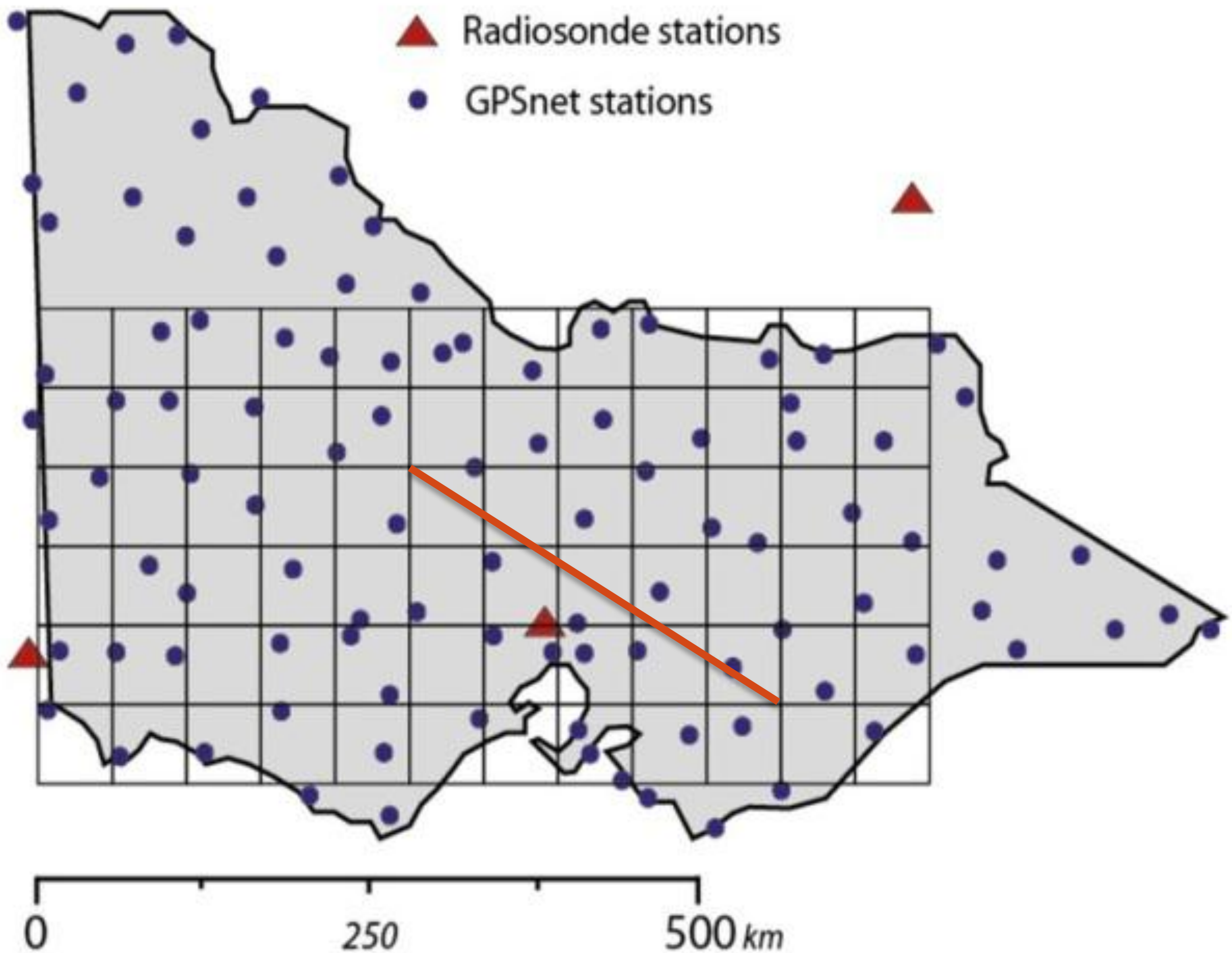
Tomography: nowcasting application (1)

6th March 2010 strong multicel storm in Melbourne, Vic, Australia

Hypothesis: 1) Strong convection in front of the severe weather should be visible in 3D tomography retrievals, 2) the rain bands removing water vapour from the air should also have signature in these data

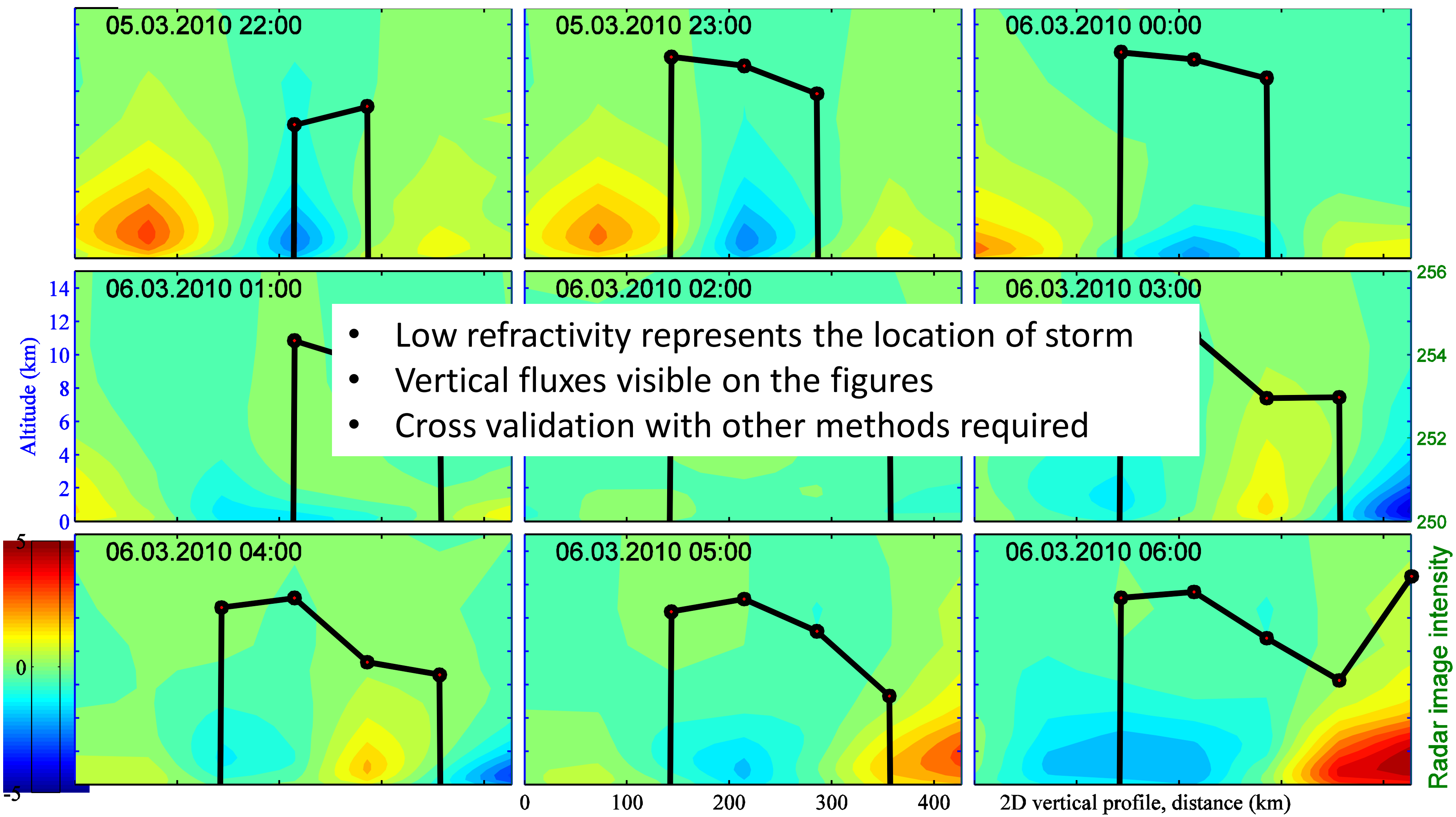


Tomography: nowcasting application (2)



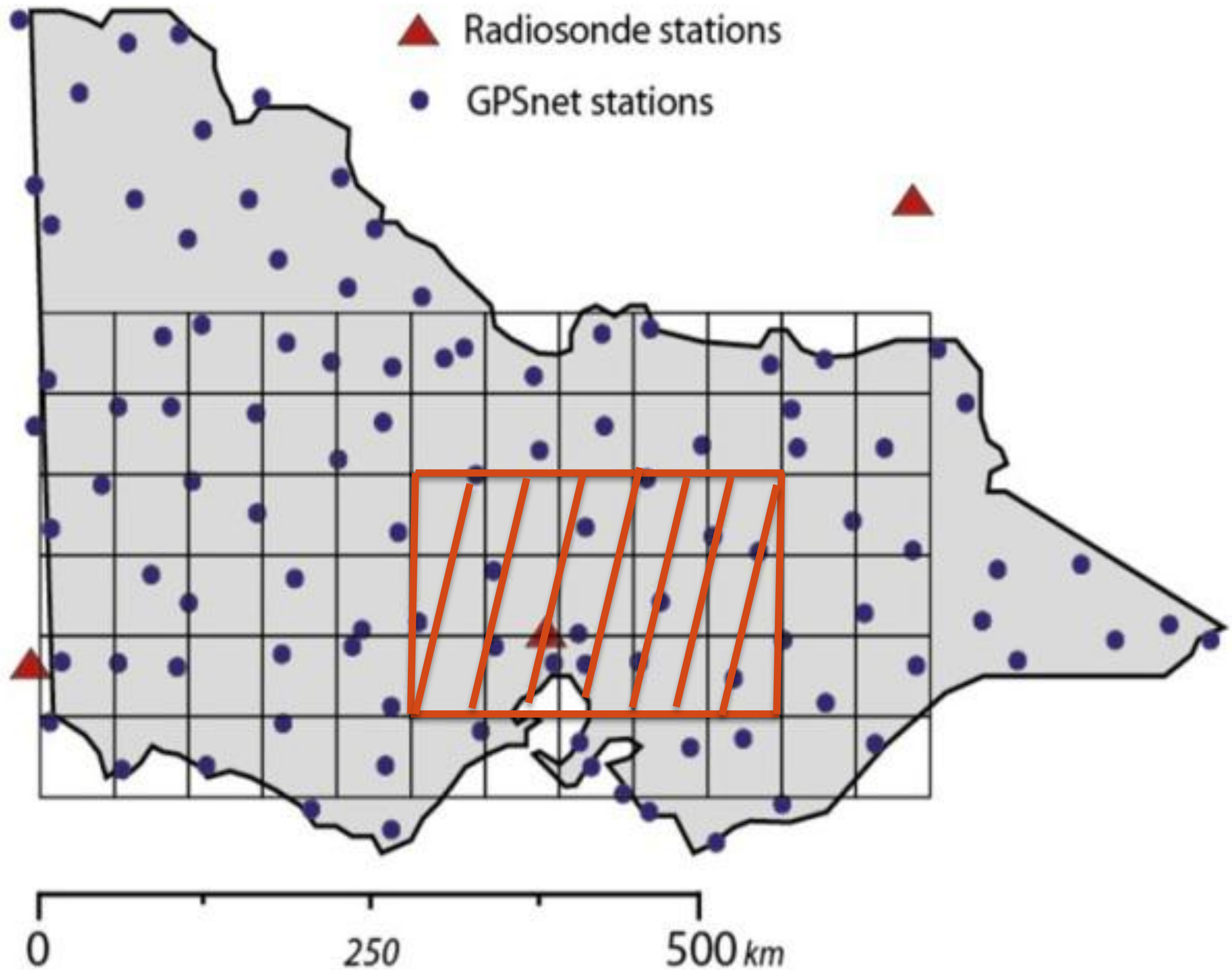
Tomography: nowcasting application (3)

Vertical cross-section of tomography model along the severe storm propagation

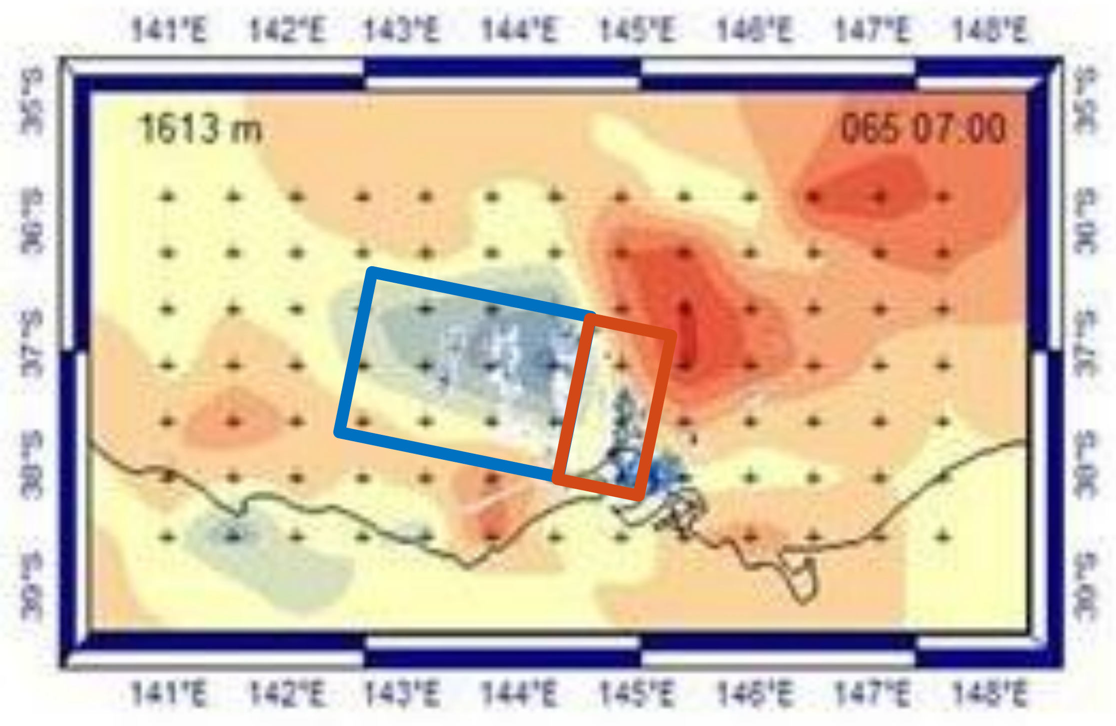


- Low refractivity represents the location of storm
- Vertical fluxes visible on the figures
- Cross validation with other methods required

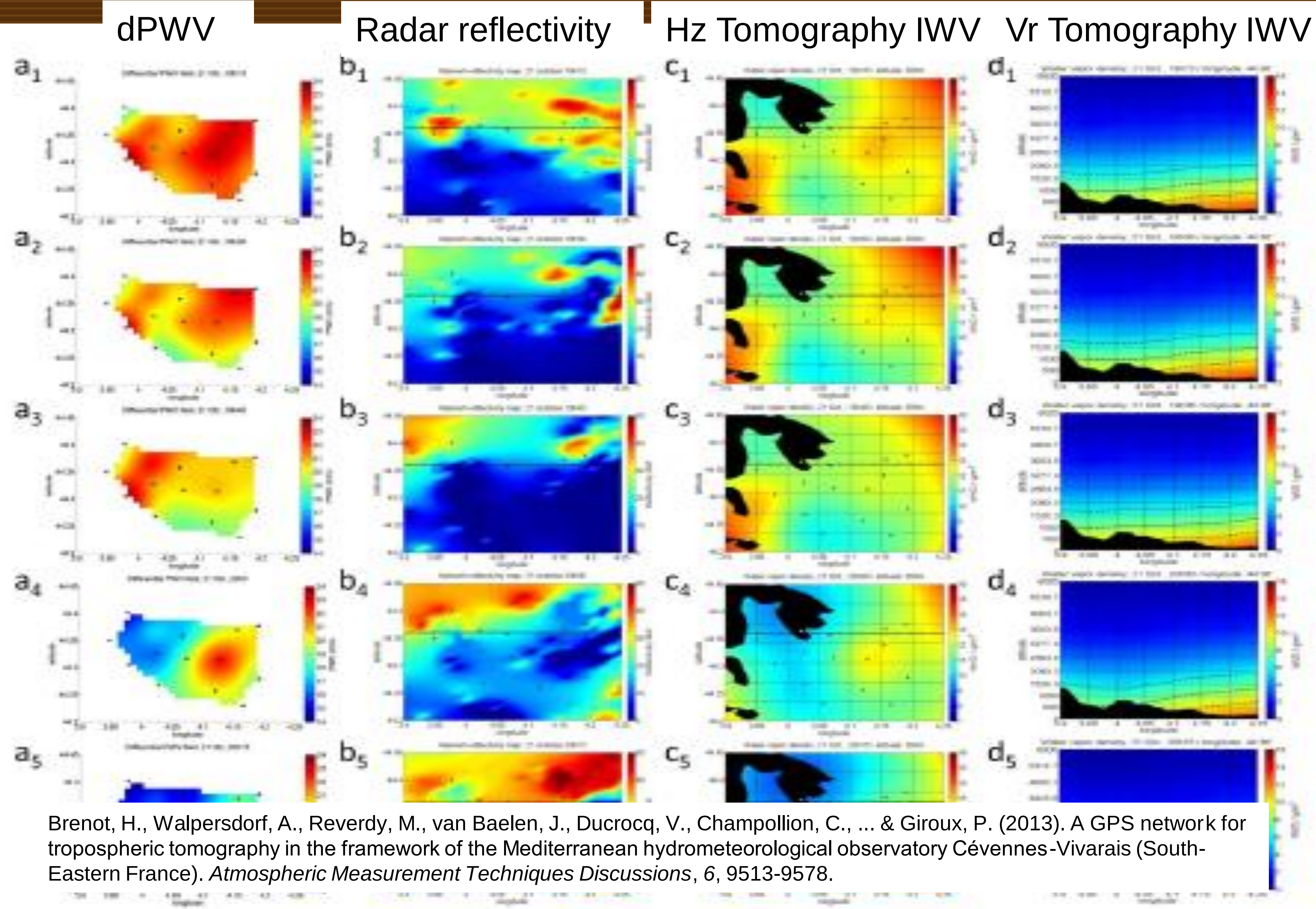
Tomography: nowcasting application (4)



Storm GNSS tomography (4)



Tormential rain: Tomography



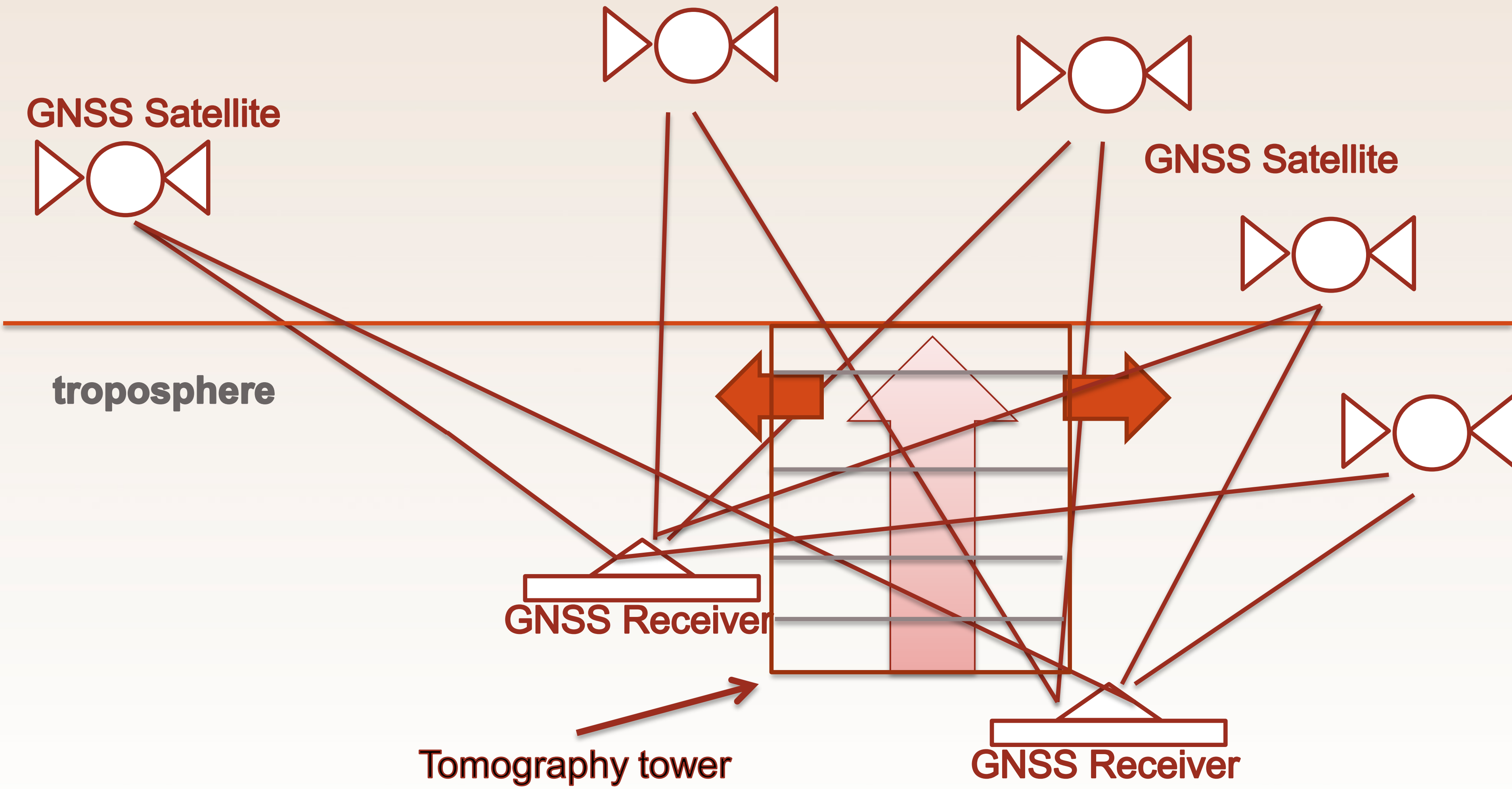
Brenot, H., Walpersdorf, A., Reverdy, M., van Baelen, J., Ducrocq, V., Champollion, C., ... & Giroux, P. (2013). A GPS network for tropospheric tomography in the framework of the Mediterranean hydrometeorological observatory Cévennes-Vivarais (South-Eastern France). *Atmospheric Measurement Techniques Discussions*, *6*, 9513-9578.

NWP tomography requirements (1)

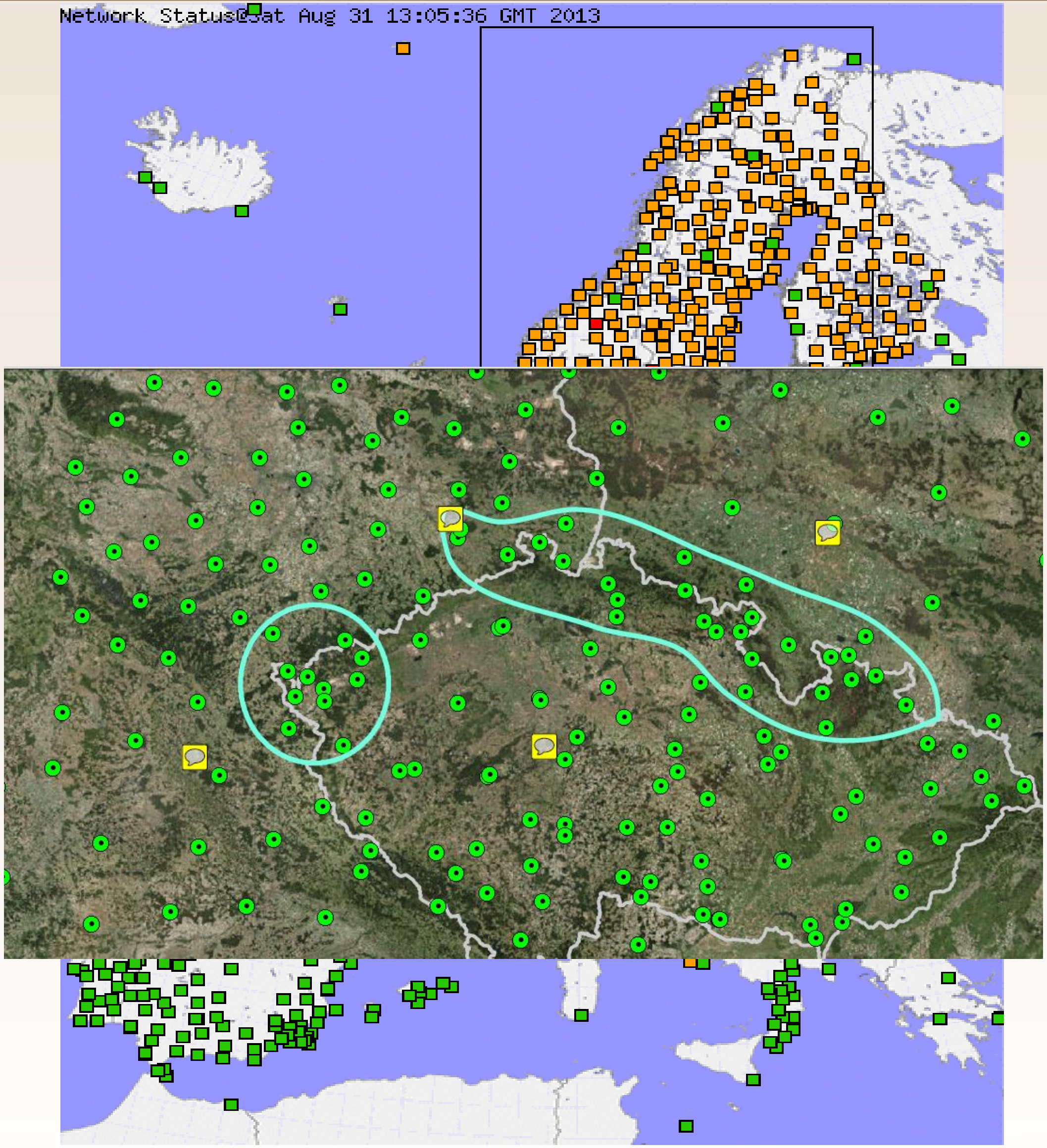
Hypothesis: The tomography profiles (fields) are easier to assimilate into the NWP system as this is not much difference to any other profiling instrument.

Number of horizontal layers	16 (20)
Bottom layer	<2.0 km (0.5km)
Top layer	6 km
Inversions	Resolved
Height difference	~300m (1000m)
Distance between receivers	<20km
Cut-off angle	5 deg (1.4 deg)
Bending impact	59mm (2397mm)

Tomography towers concept (1)



Tomography towers concept (2)



Sites that comply with requirements:

- **horizontal separation**
- **vertical separation**
- **observations separation (STD!)**
- **NRT sites**
- **stability of solution**
- **future GNSS solution**

Summary

- The tomography is a technique to convert ANY 1D observations to 3D structure
- GNSS tomography is based on: 1) the Slant Troposphere observations, 2) division of the troposphere into number of voxels and 3)know link between troposphere conditions and signal propagation
- GNSS tomography implementation for troposphere studies shoudl resolve ill-posedness of the observation system
- The quality of retrieval depends on the interstation distance, terrain undulation, available independent observations.
- There is potential to use it in both Nowcasting and NWP and we are very keen to work with you on those applications

Thank you!

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TOMOGRAPHY workshop for WG1 and WG2 in Wroclaw late Autumn 2014 (19-21 November)

- **Hands-on tomography processing of benchmark GNSS data**
- **Application to sever weather monitoring and nowcasting**