A forward operator & its adjoint for GPS Slant Total Delays

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Intro

"We developed an algorithm to compute the signal travel time delay due to the neutral atmosphere, also known as Slant Total Delay (STD), between a Global Positioning System (GPS) satellite and a ground-based station given the refractivity field of a numerical weather model. Having developed a rapid and accurate forward operator we construct the tangent-linear (adjoint) operator by application of the chain rule of differential calculus in forward (reverse) mode. Armed with these operators we show in a simulation study the potential benefit of GPS STDs in inverse modeling. We conclude that the developed operators are tailored for three (four) dimensional variational data assimilation and/or travel time tomography."

(abstract, paper submitted to Radio Science)

Further reading:

Zus et al. 2012, Radio Science.
Zus et al. 2014, Radio Science.

Non-Linear forward Model (NLM)

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Data from ray-trace comparison campaign (Nafisi et al. 2012): STDs for an elevation angle of 5° given the same NWM data (Tsukuba, Japan).

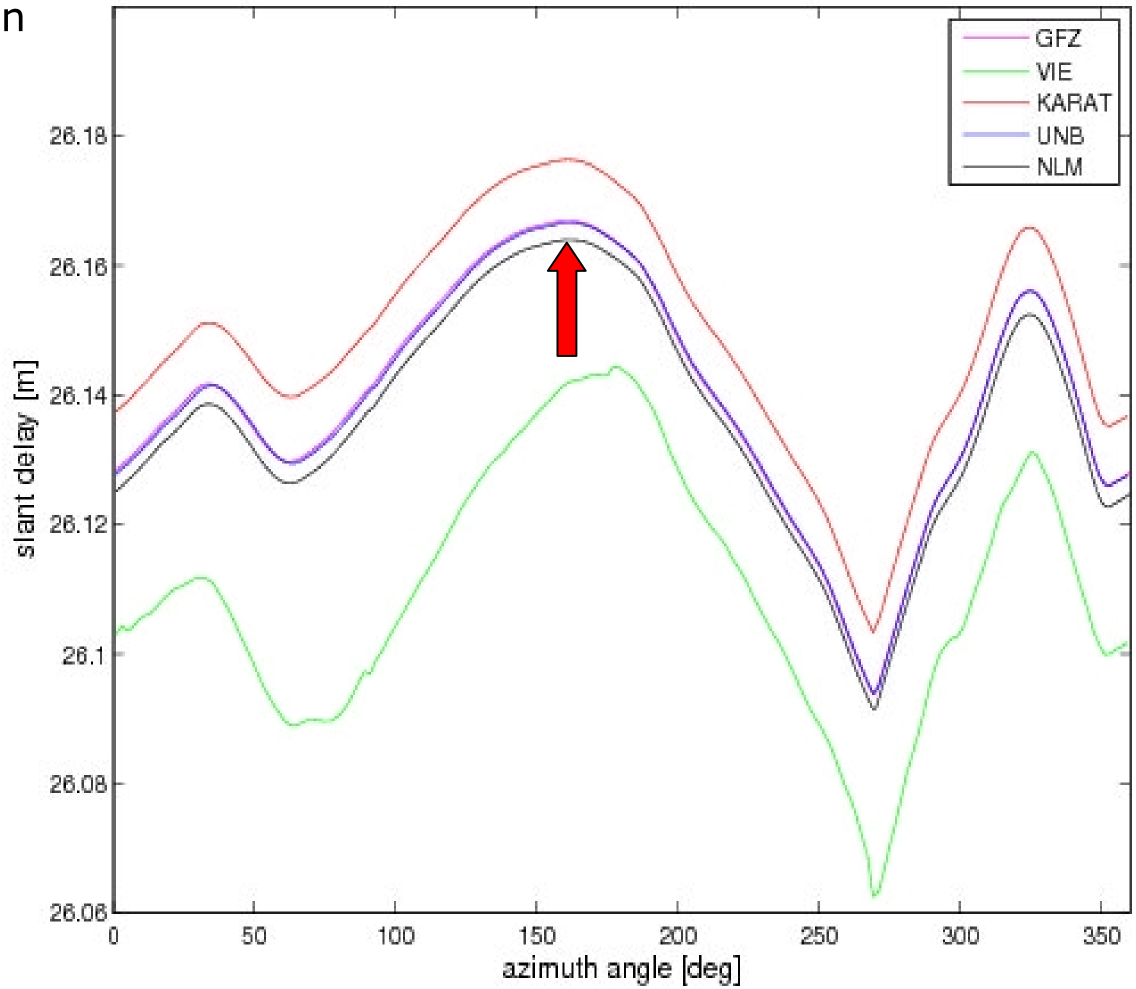
NLM accuracy:

1mm in the zenith, decreasing to 10 mm at 5°.

NLM speed:

3,000 STDs/s.

FORTRAN implementation, IFORT compiler, ordinary PC (Core2Quad Intel processor, 2.5 GHz, 2 GB RAM) and a single core (!)



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Tangent-Linear Model (TLM) & Adjoint (ADJ)

The NLM is viewed as the composite of two operators; the determination of the ray-trajectory (point-to-point ray-tracing) and the numerical quadrature:

$$H[x] = H_2 H_1 [x]$$

Method 1: Step-by-step derivation

$$H = \frac{\partial H_2}{\partial H_1} \frac{\partial H_1}{\partial x} \qquad H^t = \left(\frac{\partial H_1}{\partial x}\right)^t \left(\frac{\partial H_2}{\partial H_1}\right)^t$$

Method 2: The first-order travel time perturbation is

$$\delta S = \int \delta n \cdot ds$$

Check:

$$\lim \ \delta x \rightarrow 0 \ \frac{H[x+\delta x] - H[x]}{H \delta x} = 1 \qquad (H\delta x) \cdot \delta v = \delta x \cdot (H^t \delta v)$$

Functionality
(Inverse modeling)

Given the observations y and the (background) refractivity field *b* determine the most propable refractivity field *a*. I.e., the minimum of the cost-function

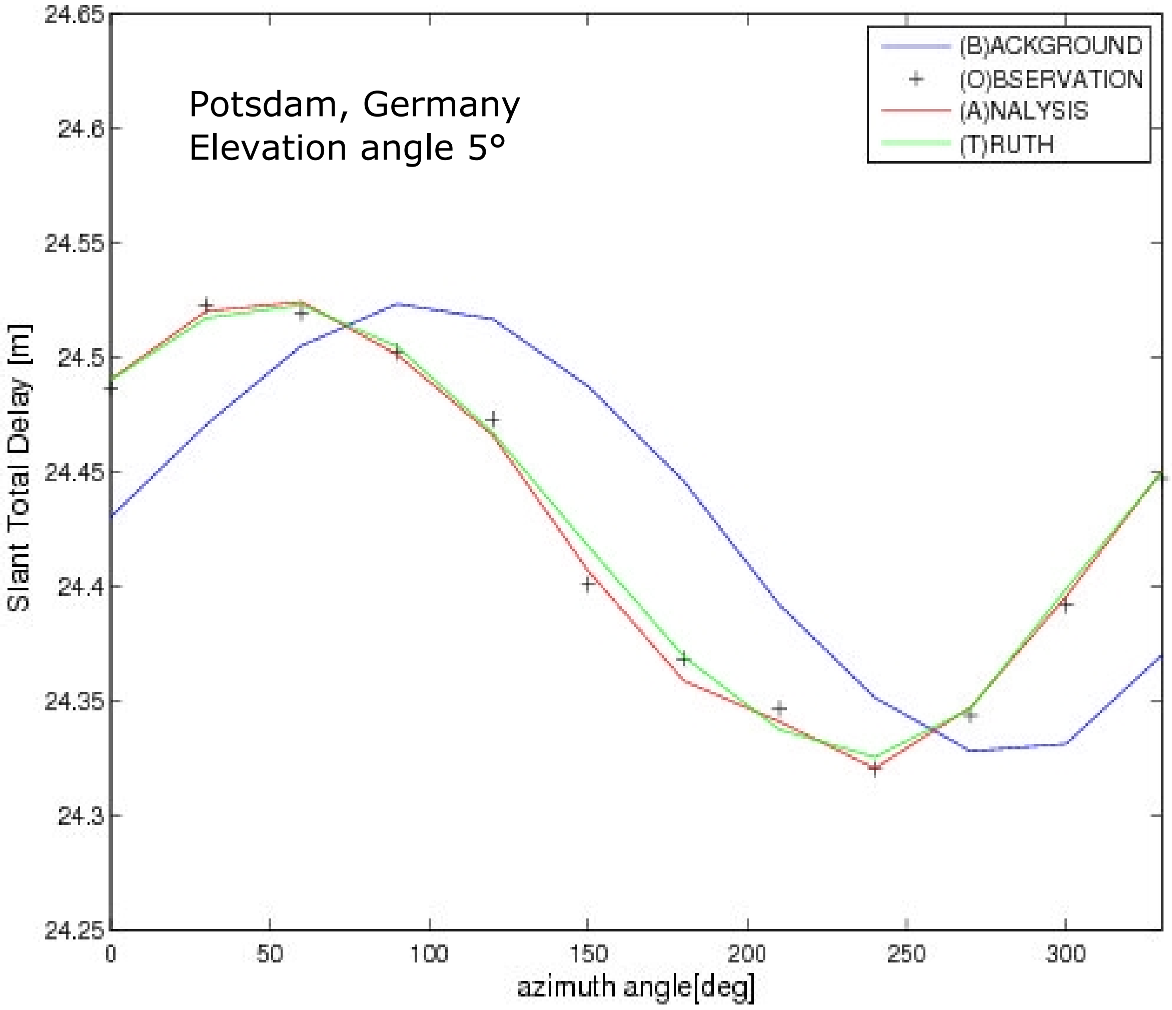
$$C[x] = \frac{1}{2}(x-b)^t B(x-b) + \frac{1}{2}(y-H[x])^t R(y-H[x])$$

must be determined. The minimum of *C* must be determined iteratively since *H* is a non-linear function of *x*. We consider a single iteration and obtain

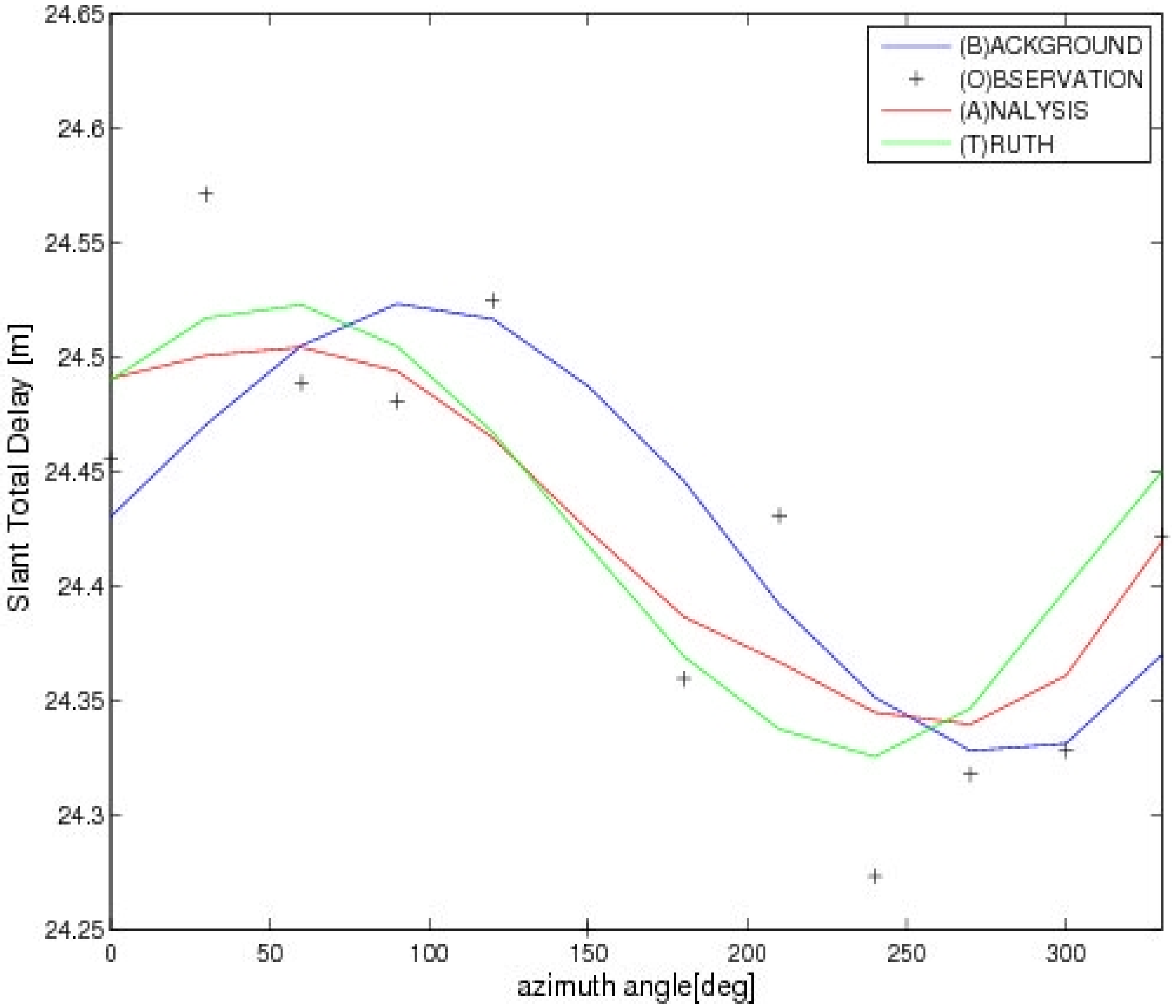
$$a = b + BH^t[b]\left(H[b]BH^t[b]+R\right)^{-1}(y-H[b])$$

Simulation:

Given a bunch of true STDs for a single station, add noise to mimic observed STDs and alter some background refractivity field such that the analysis STDs are closer to the true STDs than the background STDs. In particular, in the limit of error-free observations the analysis STDs must approach the observed (true) STDs (*y*= *H[a]*).



Case 1: perfect observations.



Case 2: imperfect observations (observation error 0.3%)

Impact

… is measured in terms of **Tropospheric Horizontal Gradient (THG)** vectors.

THG excursion

The asymmetry of STDs is approximated by [Chen and Herring, 1997]

$$\Gamma = m(e)\left[E\sin(\alpha) + N\cos(\alpha)\right]$$

N/E … North-South/East-West gradient,
m … gradient mapping function,
e/α … elevation/azimuth angle.

Given a bunch of STDs, denoted *S*, and a bunch of STDs determined under the assumption of a spherically layered refractivity field, denoted *R*, the THG vector *G = (E,N)* is obtained by

$$G = \left(\Gamma^t W^{-1} \Gamma\right)^{-1} \Gamma^t W^{-1}(S - R)$$

The weight matrix

$$W_{ij} = \sin(e_i)\sin(e_j)\delta_{ij}$$

accounts for representative errors of Γ at low elevation angles.

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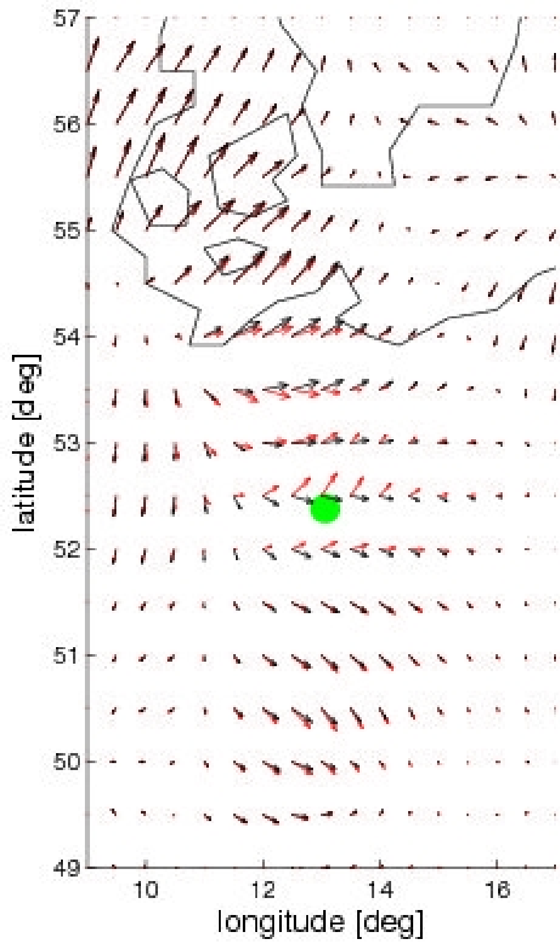
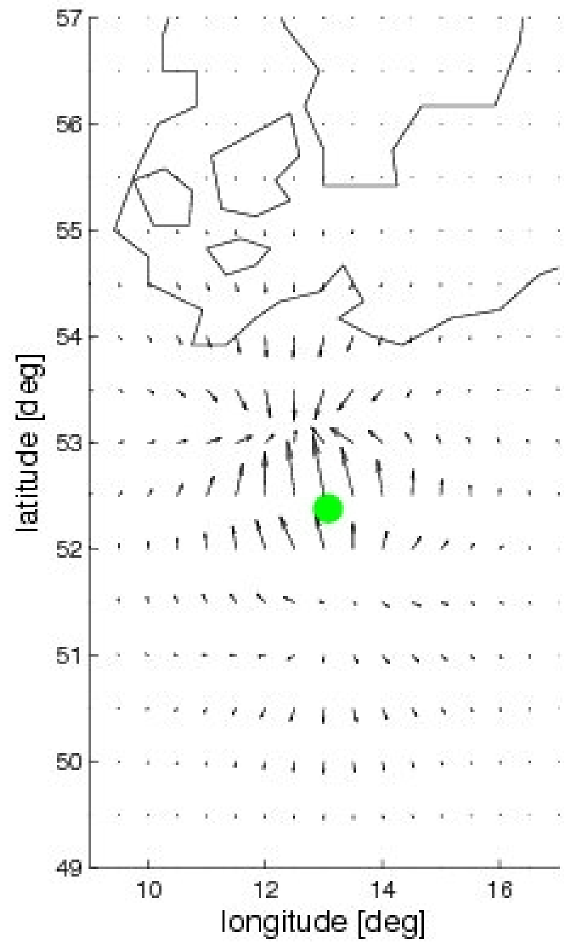
Normalized THGs:
Background (black) &
Analysis (red)



Normalized THG difference



Case 1: perfect observations.

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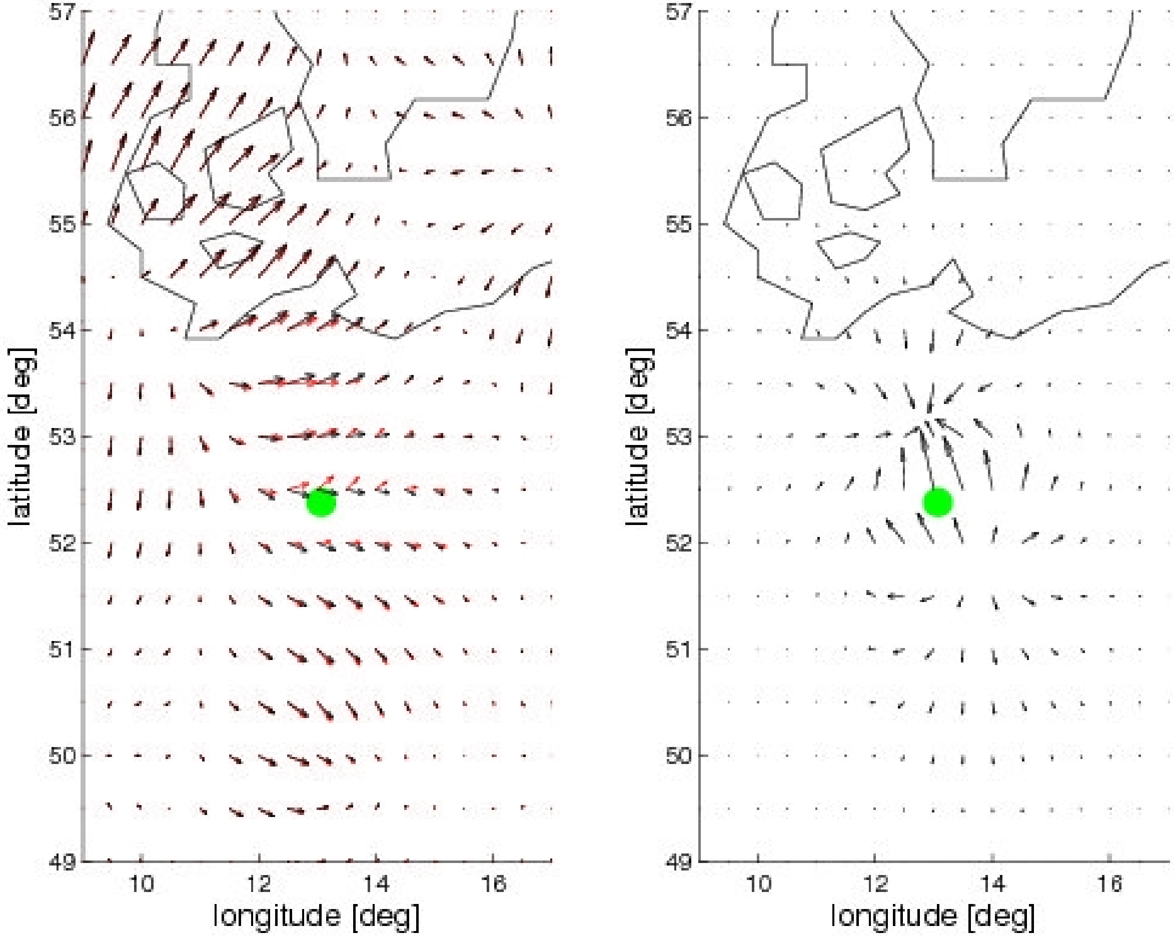

Case 2: imperfect observations.

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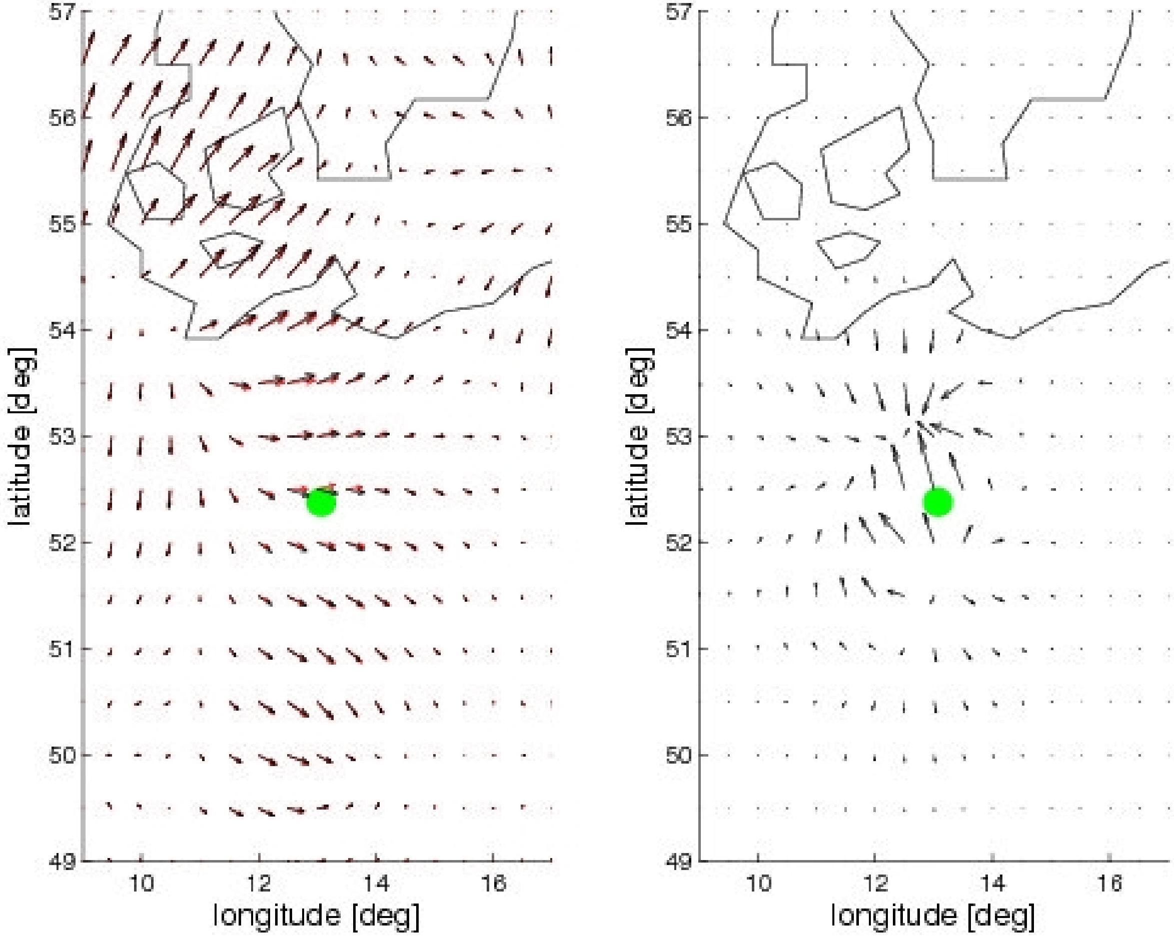

Case 3: imperfect observations & GPS satellite constellation.

Case 4: A perfect (error-free) ZTD observation has no impact.

Conclusion & Outlook

(1) A comparable NLM, TLM & ADJ does not exist.

(2) "*...In this work we performed simulation studies. We can not run conclusive assimilation studies because we do not have an operational data assimilation system. Weather agencies do have such systems but the development of the operators from scratch is time consuming. In order accelerate the exploitation of GPS STD data at weather agencies we provide the source codes upon request.*"
(conclusion, submitted to Radio Science)

(3) In that we developed a forward operator and its adjoint for STDs we developed a forward operator and its adjoint for THGs.